Formale Baumsprachen

Task 16 (Myhill-Nerode theorem)

Let $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}\}$ be a ranked alphabet and $L \subseteq T_\Sigma$ be the language consisting of all trees with exactly as many α as β symbols. Use the Myhill-Nerode theorem to show that L is not recognizable.

Task 17 (monadic second-order logic I)

Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}, \beta^{(0)}\}$ be a ranked alphabet. Consider the MSO-formula

$$\varphi = \exists U. \neg \exists x. \exists y. \operatorname{edge}_2(x, y) \wedge \operatorname{label}_\sigma(y) \wedge x \in U$$

over Σ where $x, y \in \mathcal{V}_1$ and $U \in \mathcal{V}_2$.

(a) Calculate $\operatorname{Fr}(\varphi)$ and $\operatorname{Bd}(\varphi)$ using the definitions from the lecture.

(b) Is φ closed?

Consider the tree $\xi = \sigma(\gamma(\alpha), \beta)$ and the following functions:

$$\begin{aligned}
\rho_1 &\colon\ x \mapsto \varepsilon,\ \ x' \mapsto 1,\ \ y \mapsto 11,\ \ y' \mapsto 2,\\
\rho_2 &\colon\ x \mapsto \varepsilon,\ \ x' \mapsto \varepsilon,\ \ \bar{x} \mapsto 1,\ \ y \mapsto 11,\ \ y' \mapsto 2,\\
\rho_3 &\colon\ X \mapsto \{\varepsilon, 1\},\ \ Y \mapsto \{11, 2\}, \text{ and}\\
\rho_4 &\colon\ X \mapsto \emptyset,\ \ Y \mapsto \{1, 2, 3\},\ \ x \mapsto \varepsilon.
\end{aligned}$$

(c) Which of the functions $\rho_1, ..., \rho_4$ are assignments for ξ? Give the appropriate sets of variables.

(d) Encode the assignments from Task 17 (c) as trees.

(e) Which of the trees obtained in Task 17 (d) are valid?

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