Task 14 (construction of Bar-Hillel, Perles, and Shamir)

Consider the ranked alphabet $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}, \lambda^{(0)}\}$ and the fta $A = (Q, \Sigma, \delta, F)$ where $Q = \{e, o\}$, $F = \{e\}$, and

$$\delta_\alpha = \delta_\beta = \delta_\gamma = \{(e, o)\},$$

$$\delta_\sigma = \{(q_1q_2, q_0) \in Q^2 \times Q \mid q_0 = o \iff q_1 = q_2\}.$$ 

Moreover, let us assume an fsa $B = (P, \Delta, p, G)$ where $\Delta = \Sigma^{(0)} \setminus \{\lambda\}$, $P = \{p, r\}$, $G = \{r\}$, and

$$\mu = \{(p, \alpha, p), (p, \beta, r), (r, \beta, r)\}.$$

(a) Using the technique from the lecture, construct an fta $A'$ such that

$$L(A') = L(A) \cap \text{yield}^{-1}_\lambda(L(B)).$$

(b) Are there “superfluous” transitions in $A'$ we might remove?

Task 15 (construction for $\text{Rec} \subseteq \text{Rat}$)

Consider the ranked alphabet $\Sigma = \{\alpha^{(0)}, \gamma^{(1)}\}$.

(a) Give sets $N$ and $P$ such that the regular tree grammar $G = (N, \Sigma, Z, P)$ recognizes the language

$$L = \{\xi \in T_\Sigma \mid \text{the number of occurrences of } \gamma \text{ in } \xi \text{ is not divisible by } 3\}.$$

(b) Convince yourself that $L_{Z, \delta}^N = L$ using the following definition and property:

**Definition.** For every $Q, K \subseteq N$ such that $Q \cap K = \emptyset$, and for every $A \in N$:

$$L_{A, K}^Q = \{\xi \in T_\Sigma(K) \mid \text{there is a derivation } A \Rightarrow_G \xi_1 \Rightarrow_G \ldots \Rightarrow_G \xi_n \Rightarrow_G \xi_{n+1} = \xi \text{ with } n \geq 0 \text{ such that for every } i \in [n]: \xi_i \in T_\Sigma(Q \cup K) \text{ and a rule with left-hand side in } Q \text{ is applied to } \xi_i \text{ to obtain } \xi_{i+1}\}.$$

**Property.** For every $Q, K \subseteq N$ and $A, B \in N$ such that $B \in N \setminus Q$ and $(Q \cup \{B\}) \cap K = \emptyset$:

$$L_{A, K}^{Q \cup \{B\}} = L_{A, K \cup \{B\}}^Q \cdot B \cdot (L_{B, K \cup \{B\}}^Q)^* \cdot B \cdot L_{B, K}^Q \cdot B.$$