Formale Baumsprachen

Task 7 (top-down determinism)

Let $\Delta = \{\alpha^{(0)}, \beta^{(0)}, \sigma^{(2)}\}$ be a ranked alphabet. The set $\{\sigma(\alpha, \beta), \sigma(\beta, \alpha)\} \subseteq T_\Delta$ is bottom-up deterministically recognizable but not top-down deterministically recognizable. Show that for any ranked alphabet Σ with $\Sigma^{(0)} \neq \emptyset$ and $\Sigma \neq \Sigma^{(0)} \cup \Sigma^{(1)}$ there is a language L that is bottom-up deterministically recognizable but not top-down deterministically recognizable.

Task 8 (regular tree grammars)

(a) Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$ be a ranked alphabet. Give regular tree grammars G_1 and G_2 such that

$$L(G_1) = \{\xi \in T_\Sigma \mid \xi \text{ contains exactly one } \sigma\} \text{ and}$$
$$L(G_2) = \{\xi \in T_\Sigma \mid \xi \text{ contains the pattern } \sigma(\_, \gamma(\_)) \text{ at least twice}\} .$$

(b) Let $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}\}$ be a ranked alphabet and $G = (N, \Sigma, Z, P)$ a regular tree grammar where $N = \{Z, A, B, C\}$ and

$$\begin{aligned} P = \{\ & Z \to \sigma(\sigma(A, B), C), && Z \to B, && A \to \alpha, && A \to B, \\ & B \to \beta, && B \to A, && B \to C, && C \to C\ \}\ . \end{aligned}$$

Use the construction from the lecture to give a regular tree grammar in normal form equivalent to G.

*Task 9 (*yield(Rec) $\subseteq$ CF*)*

Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}, \beta^{(0)}, \lambda^{(0)}\}$ be a ranked alphabet and $G = (N, \Sigma, Z, P)$ a regular tree grammar where $N = \{Z, A, B, C, D, E\}$ and

$$\begin{aligned} P = \{\ & Z \to \sigma(A, B), && A \to \gamma(C), && B \to \sigma(E, E), && E \to \beta, \\ & Z \to \lambda, && C \to \sigma(D, Z), && D \to \alpha\ \} \end{aligned}$$

(a) What form do the trees in $L(G)$ have? Give the languages $\text{yield}_\lambda(L(G))$ and $\text{yield}_\alpha(L(G))$.

(b) Construct a CFG G' that is λ-related to G.

*Task 10 (*CF $\subseteq$ yield(Rec)*)*

Let $\Sigma = \{[,], \langle, \rangle\}$ be an alphabet and $G = (N, \Sigma, Z, P)$ a context-free grammar where $N = \{Z\}$ and

$$P = \{Z \to ZZ, Z \to [Z], Z \to \langle Z \rangle, Z \to \varepsilon\ .\}$$

(a) Construct an equivalent CFG G' in normal form.

(b) Find a regular tree grammar H, some ranked alphabet Δ and some $e \in \Delta$ such that $\text{yield}_e(L(H)) = L(G')$.

`http://www.orchid.inf.tu-dresden.de` `Tobias.Denkinger@tu-dresden.de`