Formale Baumsprachen

**Task 7 (top-down determinism)**

Let $\Delta = \{ \alpha^{(0)}, \beta^{(0)}, \sigma^{(2)} \}$ be a ranked alphabet. The set $\{ \sigma(\alpha, \beta), \sigma(\beta, \alpha) \} \subseteq T_{\Delta}$ is bottom-up deterministically recognizable but not top-down deterministically recognizable. Show that for any ranked alphabet $\Sigma$ with $\Sigma^{(0)} \neq \emptyset$ and $\Sigma \neq \Sigma^{(0)} \cup \Sigma^{(1)}$ there is a language $L$ that is bottom-up deterministically recognizable but not top-down deterministically recognizable.

**Task 8 (regular tree grammars)**

(a) Let $\Sigma = \{ \sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)} \}$ be a ranked alphabet. Give regular tree grammars $G_1$ and $G_2$ such that

\[ L(G_1) = \{ \xi \in T_{\Sigma} \mid \xi \text{ contains exactly one } \sigma \} \]
\[ L(G_2) = \{ \xi \in T_{\Sigma} \mid \xi \text{ contains the pattern } \sigma(\_ \gamma(\_)) \text{ at least twice} \} . \]

(b) Let $\Sigma = \{ \sigma^{(2)}, \alpha^{(0)}, \beta^{(0)} \}$ be a ranked alphabet and $G = (N, \Sigma, Z, P)$ a regular tree grammar where $N = \{ Z, A, B, C \}$ and

\[ P = \{ \]
\[ Z \to \sigma(\sigma(A, B), C), \quad Z \to B, \quad A \to \alpha, \quad A \to B, \]
\[ B \to \beta, \quad B \to A, \quad B \to C, \quad C \to C \} . \]

Use the construction from the lecture to give a regular tree grammar in normal form equivalent to $G$.

**Task 9 (yield(Rec) $\subseteq$ CF)**

Let $\Sigma = \{ \sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}, \beta^{(0)}, \lambda^{(0)} \}$ be a ranked alphabet and $G = (N, \Sigma, Z, P)$ a regular tree grammar where $N = \{ Z, A, B, C, D, E \}$ and

\[ P = \{ \]
\[ Z \to \sigma(A, B), \quad A \to \gamma(C), \quad B \to \sigma(E, E), \quad E \to \beta, \]
\[ Z \to \lambda, \quad C \to \sigma(D, Z), \quad D \to \alpha \} . \]

(a) What form do the trees in $L(G)$ have? Give the languages $\text{yield}_\lambda(L(G))$ and $\text{yield}_\alpha(L(G))$.

(b) Construct a CFG $G'$ that is $\lambda$-related to $G$.

**Task 10 (CF $\subseteq$ yield(Rec))**

Let $\Sigma = \{ [ , ], \langle , \rangle \}$ be an alphabet and $G = (N, \Sigma, Z, P)$ a context-free grammar where $N = \{ Z \}$ and

\[ P = \{ \]
\[ Z \to ZZ, \quad Z \to [Z], \quad Z \to \langle Z \rangle, \quad Z \to \epsilon \} . \]

(a) Construct an equivalent CFG $G'$ in normal form.

(b) Find a regular tree grammar $H$, some ranked alphabet $\Delta$ and some $e \in \Delta$ such that $\text{yield}_e(L(H)) = L(G')$.