**Task 4 (deterministic bu-ta)**

Let $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}\}$. Give deterministic bu-ta $A_1$ and $A_2$ such that $L_1 = L(A_1)$ and $L_2 = L(A_2)$ where

(a) $L_1 = \{\xi \in T_\Sigma \mid \xi \text{ contains at least one } \alpha \text{ and one } \beta\}$
(b) $L_2 = \{\xi \in T_\Sigma \mid \xi \text{ contains an even number of } \alpha \text{ symbols}\}$

**Task 5 (finite state automata)**

Recall the concept of string automata. Let $\Sigma$ be an alphabet and $\# \notin \Sigma$. We define the ranked alphabet $\Sigma^* = \Sigma^{(0)} \cup \Sigma^{(1)}$ where $\Sigma^{(0)} = \{\#\}$ and $\Sigma^{(1)} = \Sigma$. Moreover, we define the $\Sigma^*$-algebra $(\Sigma^*, \theta)$ where $\theta(\#) = \varepsilon$ and $\theta(a)(w) = wa$ for every $a \in \Sigma$ and $w \in \Sigma^*$.

(a) Show that $\Sigma^*$ is initial in the class of $\Sigma^*$-algebras.

(b) We consider $\Sigma = \{a, b\}$ and the language $L = \{a^n b^m \mid n, m \in \mathbb{N}\}$. Sketch the diagram of a total deterministic finite-state automaton accepting $L$ and model the transition table using a finite $\Sigma^*$-algebra $Q$. How can we interpret the uniquely determined homomorphism $h: \Sigma^* \to Q$?

(c) Convince yourself that any total deterministic finite-state automaton can be modeled as a quadruple $A = (Q, \Sigma, \theta, F)$ where $(Q, \theta)$ is a finite $\Sigma^*$-algebra and $F \subseteq Q$. Define the language accepted by $A$ using the homomorphism $h: \Sigma^* \to Q$.

**Task 6 (bud-Rec(\Sigma) \subseteq Rec(\Sigma))**

Let $\Sigma$ be a ranked alphabet. In the lecture we have shown that $\text{Rec}(\Sigma)$ is a subset of $\text{bud-Rec}(\Sigma)$ using the powerset construction. Show that $\text{bud-Rec}(\Sigma)$ is a subset of $\text{Rec}(\Sigma)$. 