1st tutorial (April 22, 2015)

Formale Baumsprachen

Task 1 (definition by structural induction)

Let Σ be a ranked alphabet, $\xi, \xi_1, ..., \xi_k \in \mathrm{T}_\Sigma$, and $\zeta \in \mathrm{T}_\Sigma(X_k)$. Define the following functions by structural induction:

(a) $\mathrm{yield}(\xi)$, the sequence of leaves in ξ from left to right; and

(b) $\zeta[\xi_1, ..., \xi_k]$, the tree obtained from ζ by replacing every occurrence of x_i by ξ_i for every $i \in \{1, ..., k\}$.

In the lecture we defined trees as well-formed expressions. An alternative definition characterises a tree as a tuple (t, φ) where, intuitively, t is a set of *Gorn addresses* that is closed under certain operations and φ assigns a symbol from some alphabet Δ to every element of t.

(c) Give a formal definition of trees over Δ in the above sense.

Formally define the following characteristics of trees in the sense of Task 1 (c):

(d) height

(e) size

(f) set of positions

(g) set of subtrees

(h) label at a position

(i) subtree at a position

Task 2 (proof by structural induction)

Let Σ be a ranked alphabet and H be a set. Prove or refute the following statements for every $\xi, \zeta \in T_\Sigma(H)$, and $w \in \mathrm{pos}(\xi)$:

(a) $\xi(w) = \xi|_w(\varepsilon)$,

(b) $(\xi[\zeta]_w)|_w = \zeta$,

(c) $|\mathrm{pos}(\xi)| = |\mathrm{sub}(\xi)|$.

Task 3 (universal algebra)

(a) Recall the following concepts: Σ-algebra, Σ-homomorphism, initial Σ-algebra in a class $\mathcal{K}$, and Σ-term algebra.

(b) Show that the mappings height, size, and sub (restricted to T_Σ) are homomorphisms. Start by giving the target algebra for each of them. What is the problem concerning sub?

(c) Show that the principle of proof by structural induction is correct, applying the above concepts from universal algebra.

Note The tutorial's time might not suffice for presenting all solutions. Please prepare to ask for the solutions you are most interested in.