Formale Übersetzungsmodelle

Exercise 27 (Incomparability of TOP and BOT)

Recall the bu-tt B_1 and the td-tt T from the lecture. Prove formally that

- (a) $\tau(B_1) \notin \text{TOP}$, and
- (b) $\tau(T) \notin BOT$.

Exercise 28 (Simulation of BOT and TOP)

Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}, \beta^{(0)}\}$ be a ranked alphabet. Consider the bu-tt $B = (Q_B, \Sigma, \Sigma, F, R_B)$ and the td-tt $T = (Q_T, \Sigma, \Sigma, I, R_T)$ where $Q_B = \{*, q, q_f\}, F = \{q_f\}, Q_T = \{*, q\}, I = \{*\}$, and

$$\begin{split} R_B &= \{ \begin{array}{ll} \sigma(\ast(x_1), \ast(x_2)) \to \ast(\sigma(x_1, x_2)), & R_T &= \{ \begin{array}{ll} q(\sigma(x_1, x_2)) \to \sigma(q(x_1), q(x_2)), \\ \sigma(\ast(x_1), q(x_2)) \to q_f(x_1), & \ast(\sigma(x_1, x_2)) \to \sigma(q(x_1), \ast(x_1)), \\ \gamma(\ast(x_1)) \to \ast(\gamma(x_1)), & \ast(\sigma(x_1, x_2)) \to \sigma(\ast(x_1), q(x_1)), \\ \gamma(q(x_1)) \to q(\gamma(x_1)), & \ast(\sigma(x_1, x_2)) \to \sigma(\ast(x_1), q(x_1)), \\ \gamma(q_f(x_1)) \to q_f(\gamma(x_1)), & \ast(\gamma(x_1)) \to \gamma(\ast(x_1)), \\ \gamma(q_f(x_1)) \to q_f(\gamma(x_1)), & q(\gamma(x_1)) \to \gamma(q(x_1)), \\ \alpha \to \ast(\alpha), \ \alpha \to q(\alpha), \ \beta \to q(\beta) \ \} & \ast(\alpha) \to \alpha, \ q(\alpha) \to \alpha, \ \ast(\beta) \to \beta \ \} \end{split}$$

- (a) Identify the bottom-up and top-down specific properties of the tree transformations induced by *B* and *T* respectively.
- (b) Give td-tt T_1 and T_2 and bu-tt B_1 and B_2 such that $\tau(B) = \tau(T_1) \circ \tau(T_2)$ and $\tau(T) = \tau(B_1) \circ \tau(B_2)$.

Exercise 29 (GSM)

GSM is the class of all string transformations induced by some gsm.

- (a) Recall the formal definitions for the derivation relation of a gsm and its induced string transformation.
- (b) Prove by construction that GSM is closed under composition.
- (c) *Bonus task*: Let *G* be a gsm. Give a gsm G^R such that $\tau(G^R) = \{(v^R, w^R) | (v, w) \in \tau(G)\}$ where w^R denotes the reverse of the word *w*.