## Formale Übersetzungsmodelle

## Exercise 20 (Negation normal form)

Consider the ranked alphabet  $\Sigma = \{ \wedge^{(2)}, \vee^{(2)}, \neg^{(1)}, a^{(0)}, b^{(0)} \}$ . The trees over  $\Sigma$  represent propositional logic formulas.

- (a) Construct a td-tt *T* which brings propositional formulas into negation normal form. E.g., the negation normal form of the formula  $\neg(\neg a \lor b)$  is  $a \land \neg b$ .
- (b) Can this tree transformation also be performed by a bu-tt?

## *Exercise 21 (Symbolic derivation)*

Let  $k \in \mathbb{N}$  and  $\Sigma = \{+^{(2)}, \cdot^{(2)}, X^{(0)}\} \cup \{0^{(0)}, \dots, k^{(0)}\}$  be a ranked alphabet. The trees  $\xi$  over  $\Sigma$  represent certain polynomials with natural number coefficients.

- (a) Give a td-tt *T* which computes the symbolic derivation  $d\xi/dx$  of a given polynomial. E.g., the symbolic derivation of the polynomial  $(X \cdot X + 5) \cdot X$  is  $((1 \cdot X + X \cdot 1) + 0) \cdot X + (X \cdot X + 5) \cdot 1$ , using the well-known sum and product rules from calculus.
- (b) Give a derivation of  $\xi = \cdot (+(\cdot(X,X),5),X)$  in *T*.
- (c) Can this tree transformation also be performed by a bu-tt?
- (d) *Extra task:* Can we model (natural number) exponents, or other more interesting operations like sin(*x*), exp(*x*)?

## Exercise 22 (Subclasses of TOP)

Describe the relations of the classes of tree transformations induced by the following transducers:

- td-tt,
- deterministic td-tt,
- total td-tt,
- total and deterministic td-tt,
- top-down tree homomorphisms,
- top-down relabelings,
- linear td-tt,
- nondeleting td-tt,
- and linear and nondeleting td-tt.

Use the abbreviations TOP, *d*-TOP, *t*-TOP, *dt*-TOP, *h*-TOP, *r*-TOP, *l*-TOP, *n*-TOP, and *ln*-TOP.