2nd Exercise Sheet (October 23, 2014)

Formale Übersetzungsmodelle

Exercise 4 (Structural induction, revisited)

Let Σ be a ranked alphabet. Prove or refute: There is an integer $c \geq 1$ such that, for every $\xi \in T_\Sigma(X)$, $\text{size}(\xi) \leq c^{\text{height}(\xi)}$.
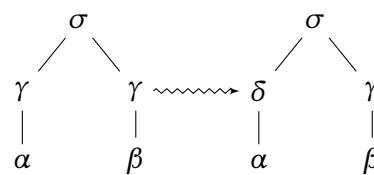
Exercise 5 (Bottom-up tree transducers)

Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$.

(a) Give a bu-tt M_1 that, for every tree $\xi \in T_\Sigma$, annotates every position $w \in \text{pos}(\xi)$ with the last symbol in w (if any).

(b) Give a bu-tt M_2 that, for every tree $\xi \in T_\Sigma$, replaces the first occurrence (according to pre-order traversal) of γ in ξ by $\delta^{(1)}$, without changing the rest of ξ.

(c) Let $\Gamma = \{\delta^{(2)}, \alpha^{(0)}\}$ and $\Delta = \Sigma \cup \Gamma$. Give a bu-tt M_3 which performs the following tree transformation: Let $\xi \in T_\Delta$. For every $w \in \text{pos}(\xi)$ such that $\xi|_w \in T_\Gamma$ and

 (i) either $w = \epsilon$

 (ii) or $w = vi$ for some $i \in \mathbb{N}$ and $v \in \mathbb{N}^*$ with $\xi|_v \notin T_\Gamma$,

the subtree $\xi|_w$ of ξ is replaced by the tree $\gamma^{n-1}(\alpha)$, where n is the length of the leftmost branch of $\xi|_w$. The remainder of ξ is to be left unchanged.

(d) *Bonus question:* Implement the same transformation as in (c), but with $\xi|_w$ replaced by $\sigma(\gamma^{n-1}(\alpha), \gamma^{n-1}(\alpha))$ instead of $\gamma^{n-1}(\alpha)$.
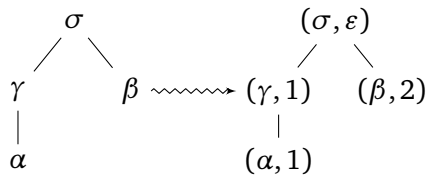
(a) transformation $\tau(M_1)$ (b) transformation $\tau(M_2)$
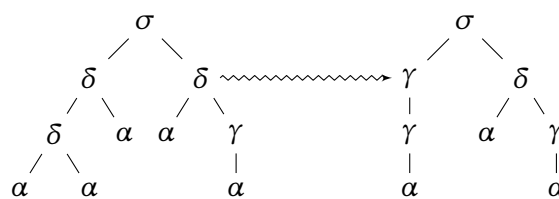
(c) transformation $\tau(M_3)$

Exercise 6 (Output-height bound)

(a) Let $M = (Q, \Sigma, \Delta, F, R)$ be a bu-tt. Prove that there is some $c \in \mathbb{N}$ such that for every $(\xi, \zeta) \in \tau(M)$, we have $\text{height}(\zeta) \leq c \cdot \text{height}(\xi)$.

(b) Let $\Sigma = \{\gamma^{(1)}, \alpha^{(0)}\}$. Prove or refute: There is a bu-tt M such that $\tau(M) = \{(\gamma^n(\alpha), \gamma^{2^n}(\alpha))\}$.