

Parsing of Natural Languages

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joint work with

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[Chomsky 56]: Three Models for the Description of Language.

*"There are **two central problems** in the descriptive study of languages. One primary concern of the linguist is **to discover simple and *revealing*** grammars for natural languages. At the same time, **by studying the properties of such successful grammars ...**, he hopes to arrive at a general theory of linguistic structure."*

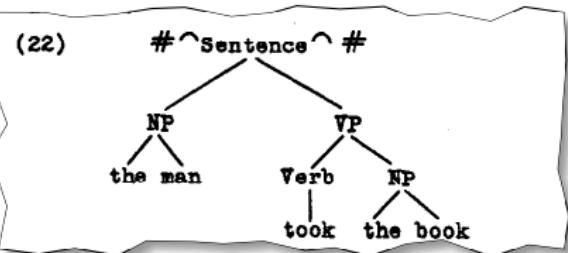
[Chomsky 56]: Three Models for the Description of Language.

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context-free grammar:

(20) $\Sigma : \# \wedge \text{Sentence} \wedge \#$
F: Sentence \rightarrow NP VP
VP \rightarrow Verb NP
NP \rightarrow the man, the book
Verb \rightarrow took

phrase structure tree:



E : natural language \mathcal{H} : set of syntactic structures

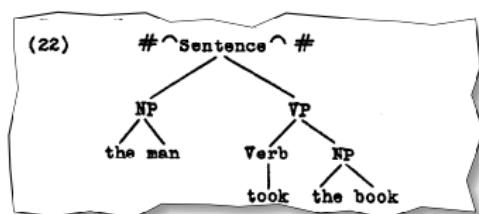
parsing : $E \rightarrow \mathcal{H}$

E : natural language \mathcal{H} : set of syntactic structures

$$\text{parsing} : E \rightarrow \mathcal{H}$$

parsing with context-free grammar G :

parsing(the man took the book) =



Outline

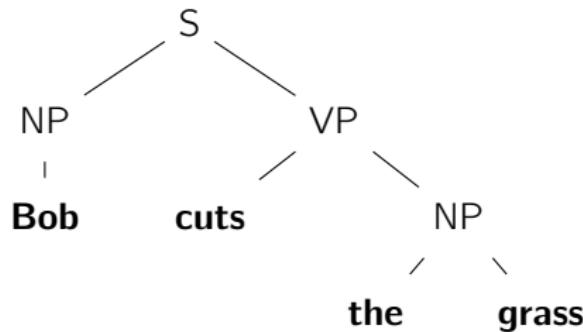
- ▶ Syntax of natural language sentences
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- ▶ A particular “simple and revealing grammar” model
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Bob 1 **cuts** 2 **the** 3 **grass** 4

phrase structure tree:



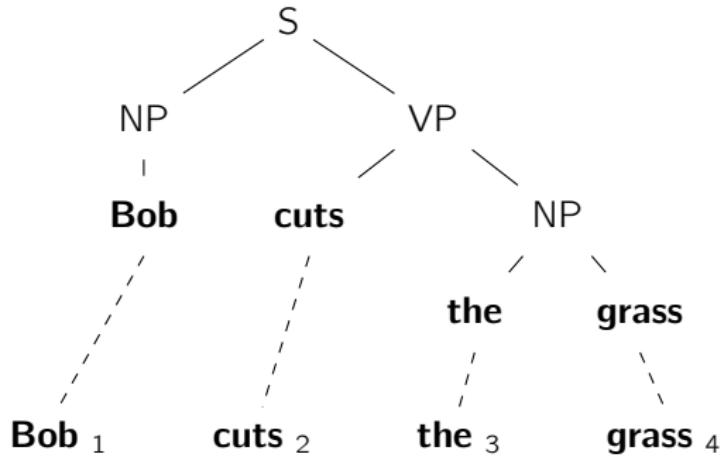
Bob 1

cuts 2

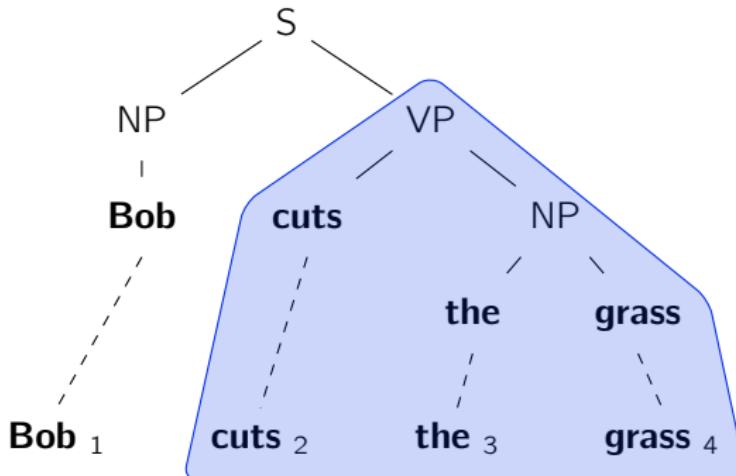
the 3

grass 4

phrase structure tree:



phrase structure tree: (continuous)



continuous:

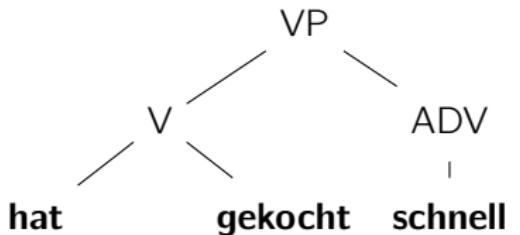
each tree position covers an interval of sentence positions

hat 1
(has)

schnell 2
(fast)

gekocht 3
(cooked)

phrase structure tree:

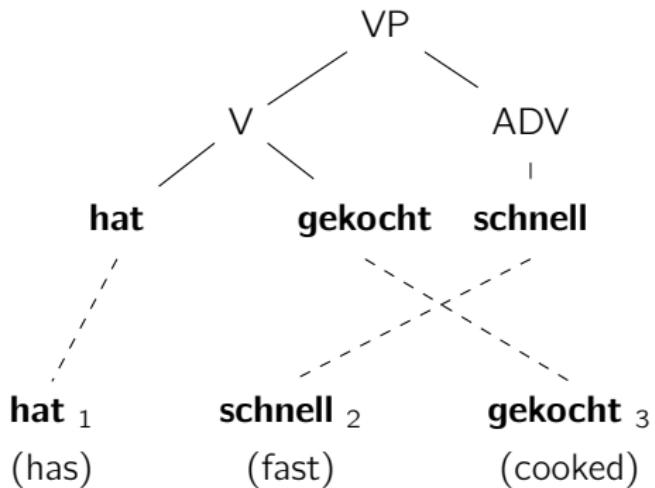


hat ₁
(has)

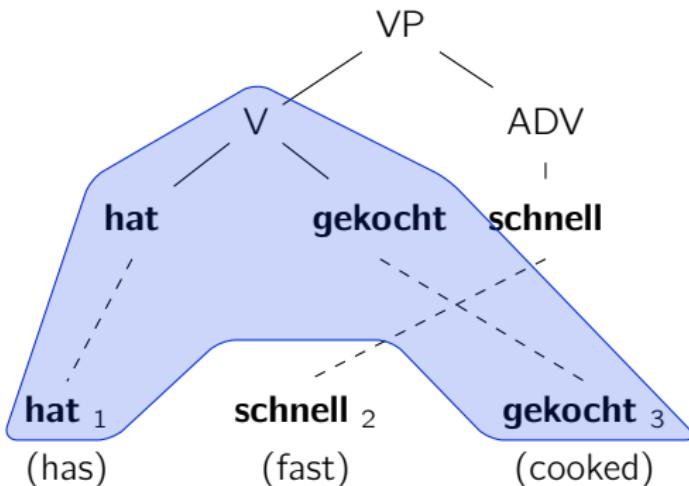
schnell ₂
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gekocht ₃
(cooked)

phrase structure tree:



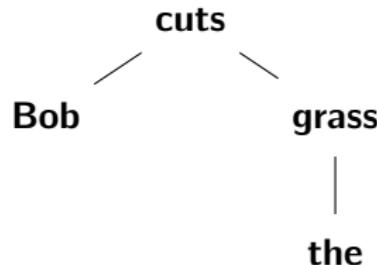
phrase structure tree: (discontinuous)



Bob 1 **cuts** 2 **the** 3 **grass** 4

dependency tree:

[Tesnière 59]



Bob 1

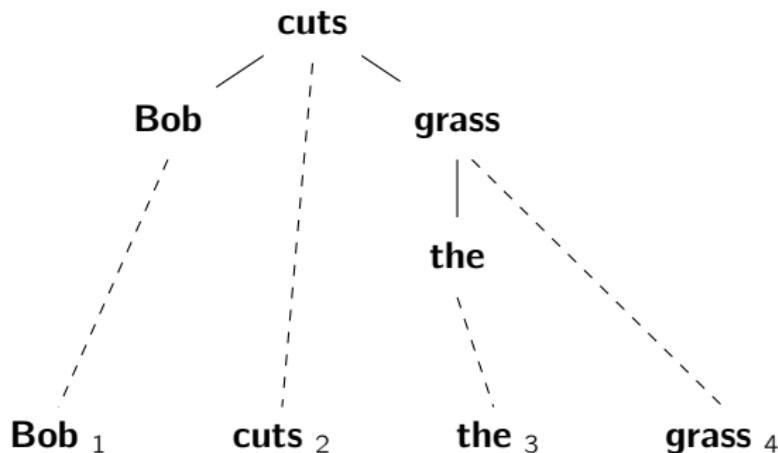
cuts 2

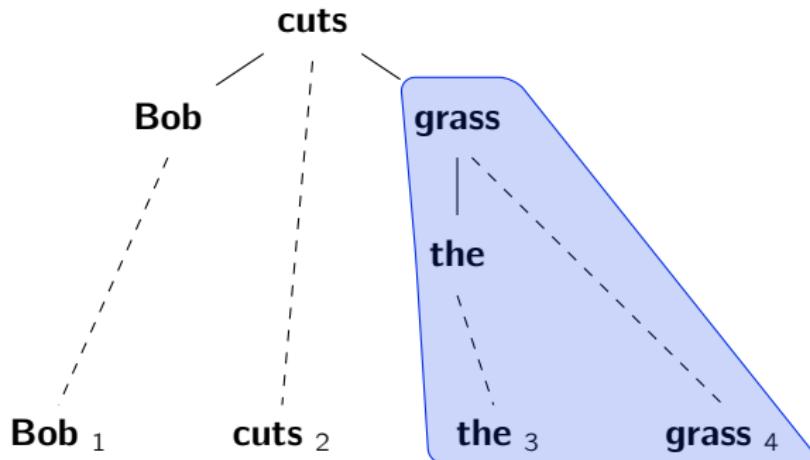
the 3

grass 4

dependency tree:

[Tesnière 59]



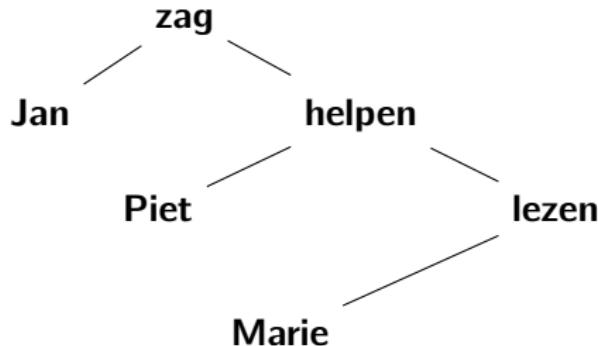


projective:

each tree position covers an interval of sentence positions

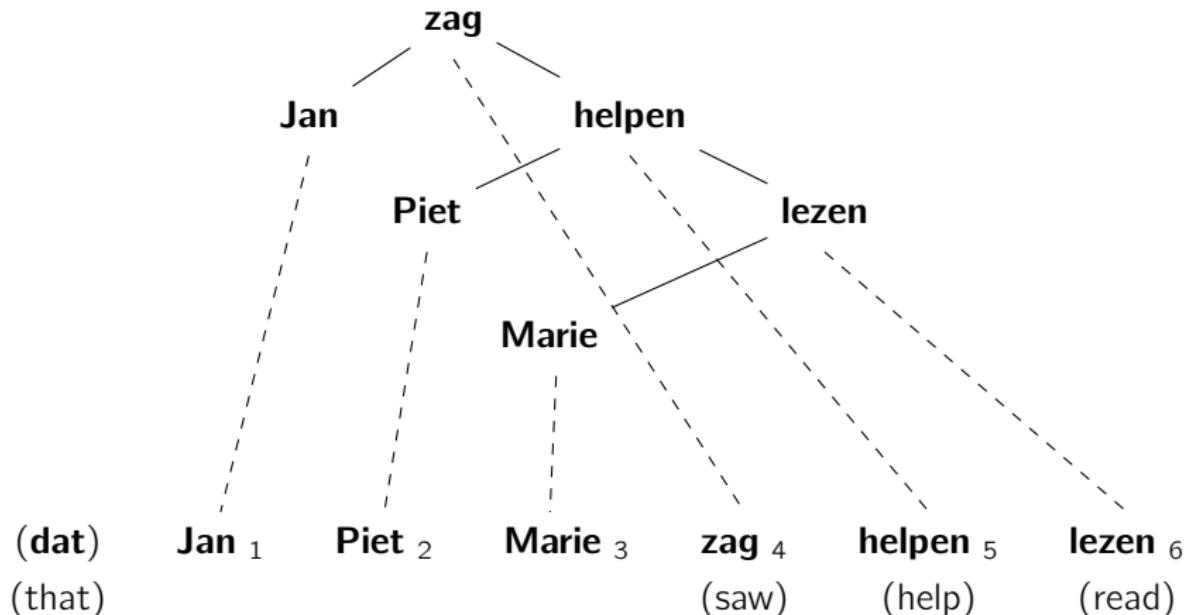
(dat) Jan 1 Piet 2 Marie 3 zag 4 helpen 5 lezen 6
(that) (saw) (help) (read)

dependency tree:

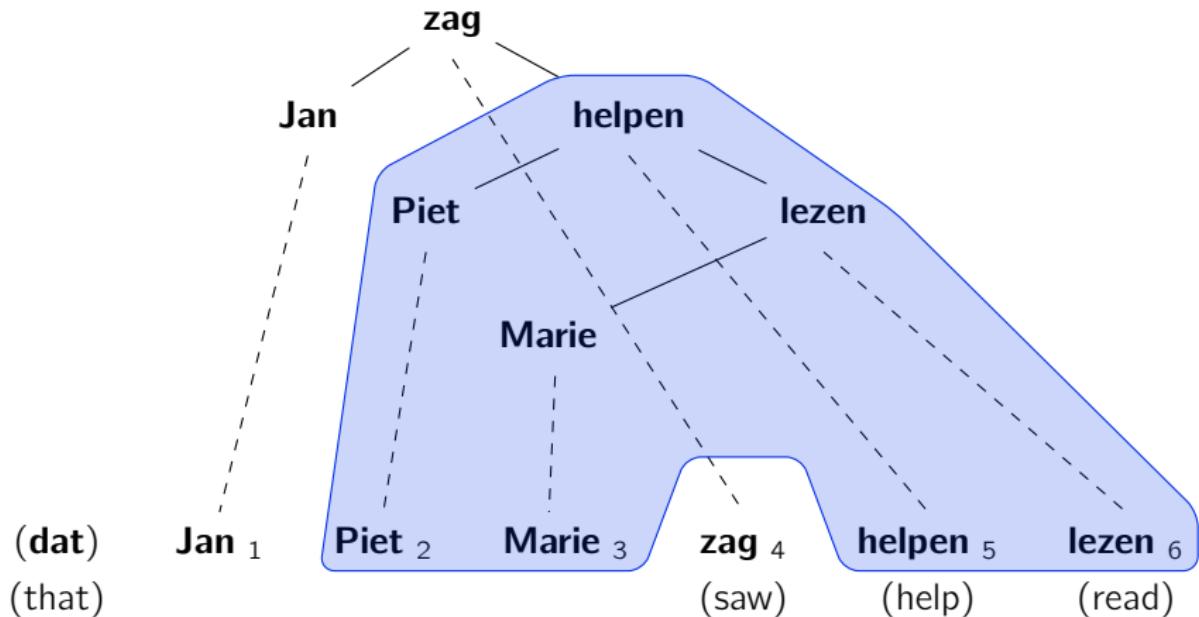


(dat)	Jan 1	Piet 2	Marie 3	zag 4	helpen 5	lezen 6
(that)				(saw)	(help)	(read)

dependency tree:



dependency tree: (non-projective)



Let Σ be a ranked alphabet. A hybrid tree is a tuple $h = (\xi, U, \preceq)$ where

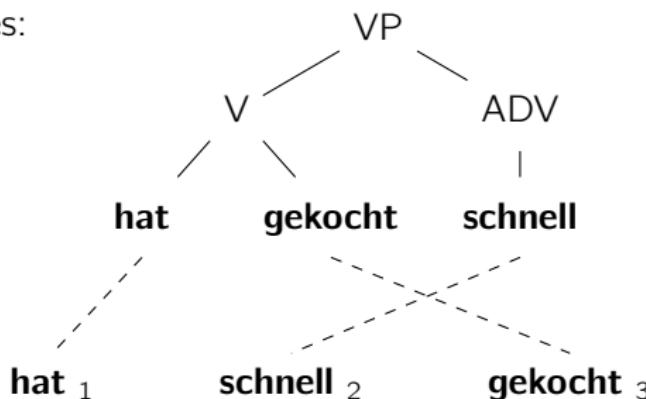
- ▶ ξ is a tree over Σ
- ▶ $U \subseteq \text{pos}(\xi)$
- ▶ \preceq linear order on U

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phrase structure trees:

$U = \text{leaves}(\xi)$

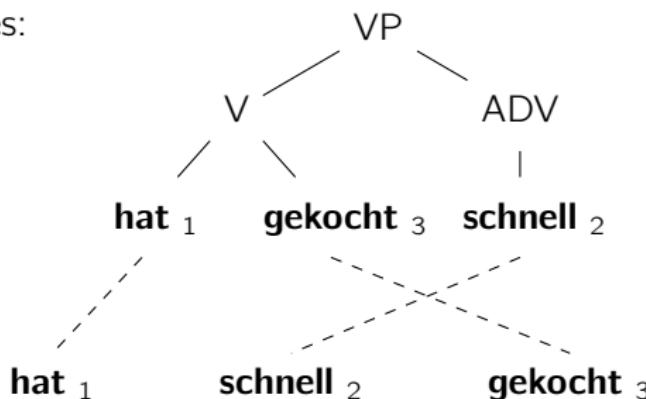


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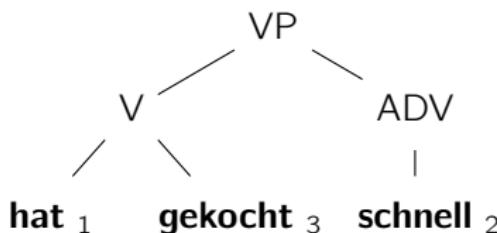


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phrase structure trees:

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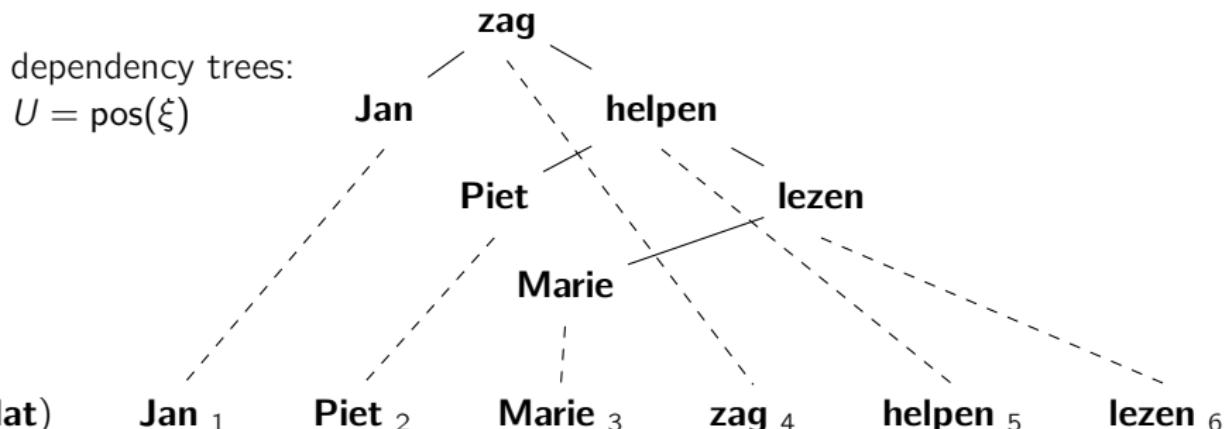


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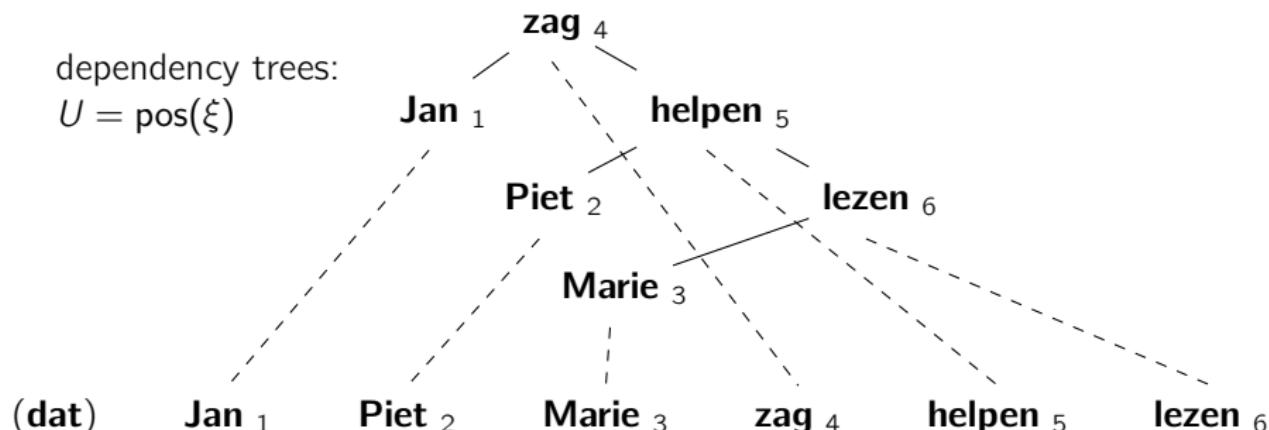


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dependency trees:

$$U = \text{pos}(\xi)$$

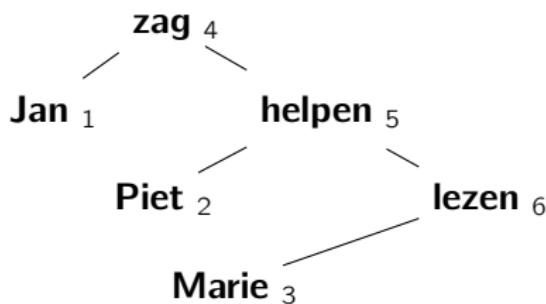


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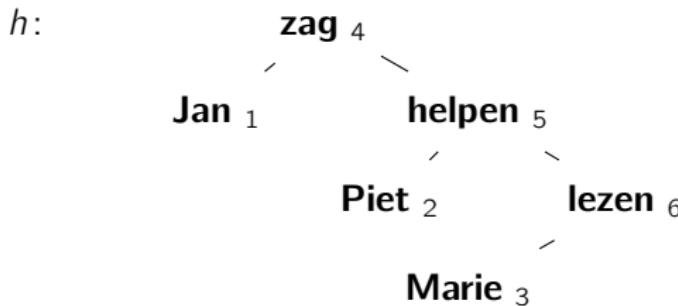


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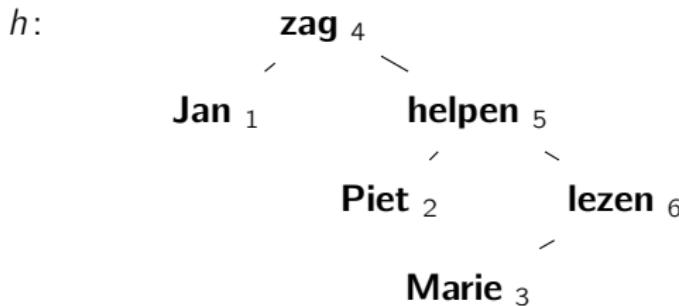
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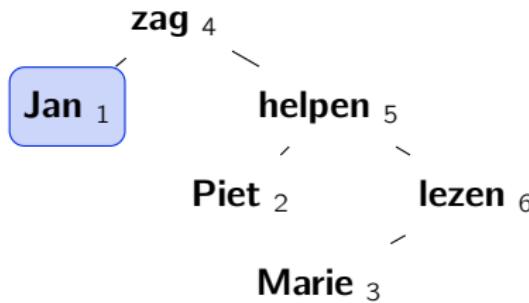


$\text{str}(h) =$

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$h:$

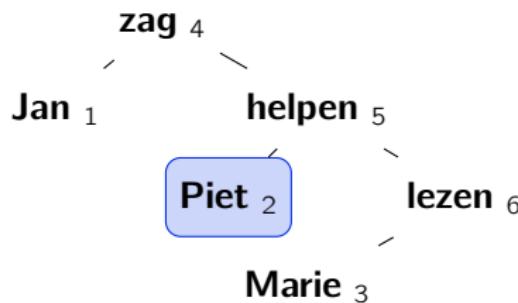


$\text{str}(h) =$ **Jan** ₁

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$h:$

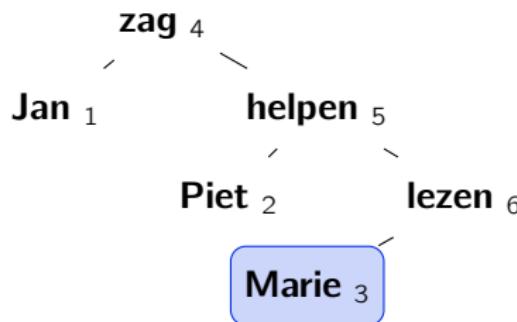


$$\text{str}(h) = \text{Jan}_1 \text{ Piet}_2$$

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$h:$

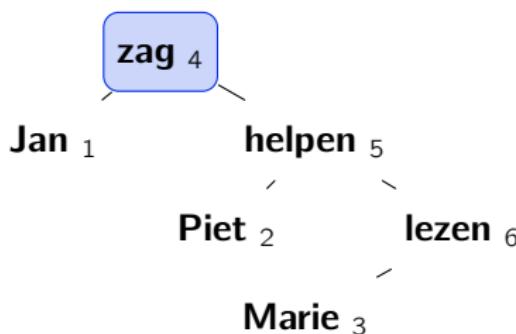


$$\text{str}(h) = \text{Jan } 1 \text{ Piet } 2 \text{ Marie } 3$$

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$h:$

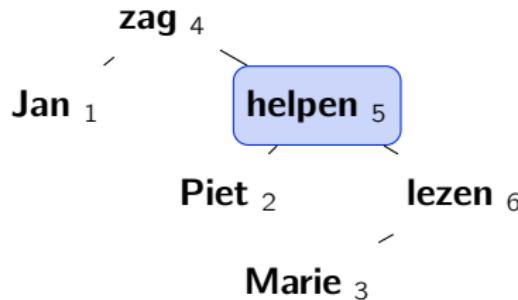


$$\text{str}(h) = \text{Jan } 1 \text{ Piet } 2 \text{ Marie } 3 \text{ zag } 4$$

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$h:$

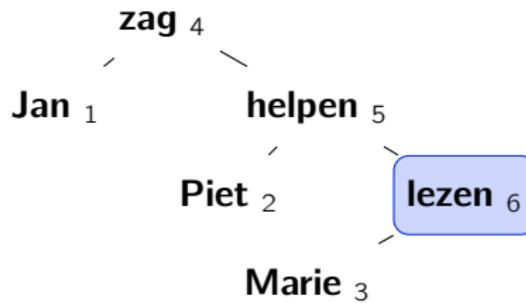


$$\text{str}(h) = \text{Jan } 1 \text{ Piet } 2 \text{ Marie } 3 \text{ zag } 4 \text{ helpen } 5$$

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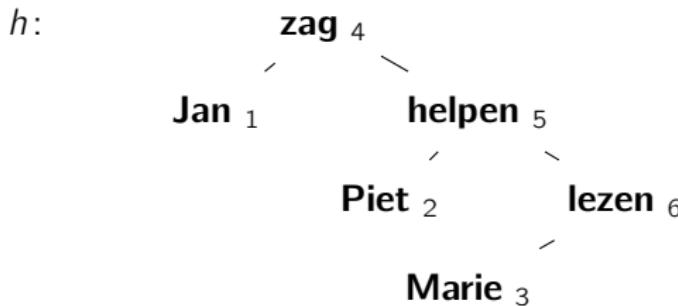
$h:$



$\text{str}(h) = \text{Jan } 1 \text{ Piet } 2 \text{ Marie } 3 \text{ zag } 4 \text{ helpen } 5 \text{ lezen } 6$

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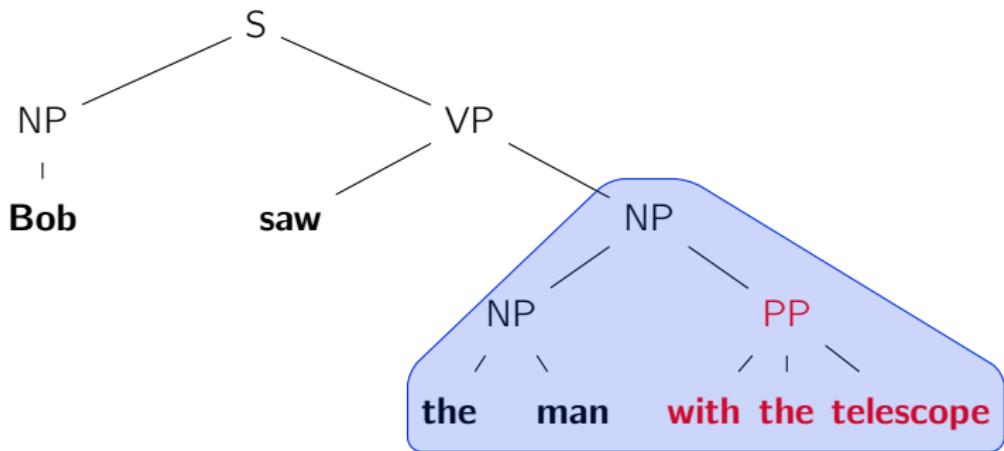
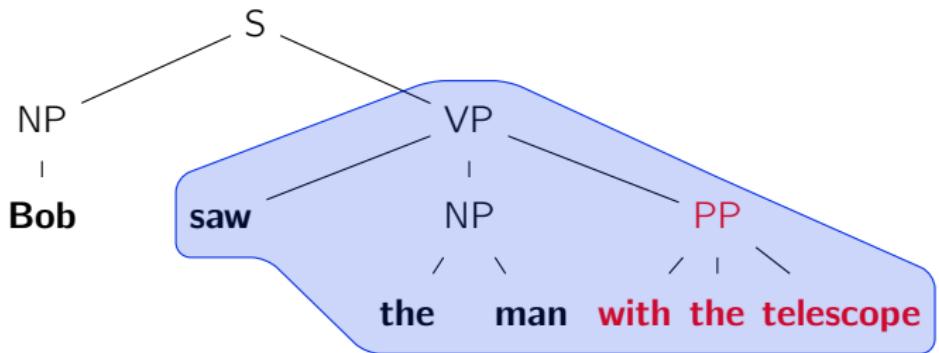
$$\text{str}(h) = \text{Jan } 1 \text{ Piet } 2 \text{ Marie } 3 \text{ zag } 4 \text{ helpen } 5 \text{ lezen } 6$$

Outline

- ▶ Syntax of natural language sentences
 - ▶ hybrid trees: phrase structure trees, dependency trees
- ▶ Ambiguity and parsing
- ▶ A particular “simple and revealing grammar” model
- ▶ Probabilistic hybrid grammars

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E : natural language \mathcal{H} : set of syntactic structures

parsing : $E \rightarrow \mathcal{H}$

parsing(e) = argmax _{$h \in \mathcal{H}: \text{str}(h) = e$} $P(h | \mathcal{M})$

where

- ▶ \mathcal{M} : probabilistic language model
- ▶ $P(h | \mathcal{M})$: probability of h given \mathcal{M}

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[Vijay-Shanker, Weir, Joshi 87]
[Seki, Matsumura, Fujii, Kasami 91]

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LCFRS G :

S	\rightarrow	N	V
V	\rightarrow	N	V
V	\rightarrow	N	
N	\rightarrow	ϵ	
N	\rightarrow	ϵ	
N	\rightarrow	ϵ	

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LCFRS G :

$$\begin{array}{ll} S & \rightarrow N(x) \ V(y_1, y_2) \\ V & \rightarrow N(x) \ V(y_1, y_2) \\ V & \rightarrow N(x) \\ N & \rightarrow \varepsilon \\ N & \rightarrow \varepsilon \\ N & \rightarrow \varepsilon \end{array}$$

$$\text{fanout}(S) = \text{fanout}(N) = 1$$

$$\text{fanout}(V) = 2$$

probabilistic linear context-free rewriting systems (prob. LCFRS) (G, p)
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LCFRS G :

$$S(x \ y_1 \ \mathbf{zag} \ y_2) \rightarrow N(x) \ V(y_1, y_2)$$

$$V(x \ y_1, \ \mathbf{helpen} \ y_2) \rightarrow N(x) \ V(y_1, y_2)$$

$$V(x, \ \mathbf{lezen}) \rightarrow N(x)$$

$$N(\mathbf{Jan}) \rightarrow \varepsilon$$

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no copying/deletion of variables

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LCFRS G :

$S(x \ y_1 \ \mathbf{zag} \ y_2)$	\rightarrow	$N(x) \ V(y_1, y_2)$	1
$V(x \ y_1, \ \mathbf{helpen} \ y_2)$	\rightarrow	$N(x) \ V(y_1, y_2)$	0.5
$V(x, \ \mathbf{lezen})$	\rightarrow	$N(x)$	0.5
$N(\mathbf{Jan})$	\rightarrow	ε	0.33
$N(\mathbf{Piet})$	\rightarrow	ε	0.33
$N(\mathbf{Marie})$	\rightarrow	ε	0.33

probability assignment p :

$$\begin{aligned}\text{fanout}(S) &= \text{fanout}(N) = 1 \\ \text{fanout}(V) &= 2\end{aligned}$$

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probability assignment p :

1

0.5

0.5

0.33

0.33

0.33

$$\text{fanout}(S) = \text{fanout}(N) = 1$$

$$\text{fanout}(V) = 2$$

no copying/deletion of variables
lexicalized LCFRS ("anchor")

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LCFRS G : probability assignment p :

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$V(x \ y_1, \ \mathbf{helpen} \ y_2)$	$\rightarrow N(x) \ V(y_1, y_2)$	0.5
$V(x, \ \mathbf{lezen})$	$\rightarrow N(x)$	0.5
$N(\mathbf{Jan})$	$\rightarrow \varepsilon$	0.33
$N(\mathbf{Piet})$	$\rightarrow \varepsilon$	0.33
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no copying/deletion of variables
 lexicalized LCFRS ("anchor")

CFG = LCFRS(fanout=1):

$$A \rightarrow aBbC \rightsquigarrow A(a \ x_1 \ b \ x_2) \rightarrow B(x_1) \ C(x_2)$$

lexicalized LCFRS G :

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$$S(\mathbf{J}_1 \ \mathbf{P}_2 \ \mathbf{M}_3 \ \mathbf{z}_4 \ \mathbf{h}_5 \ \mathbf{t}_6)$$

lexicalized LCFRS G :

$$S(x \ y_1 \ \mathbf{zag} \ y_2) \rightarrow N(x) \ V(y_1, y_2)$$

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lexicalized LCFRS G :

$$S(\boxed{x} \; \boxed{y_1} \; \text{zag} \; \boxed{y_2}) \rightarrow N(\boxed{x}) \; V(\boxed{y_1}, \boxed{y_2})$$

$$V(x \; y_1, \; \text{helpen} \; y_2) \rightarrow N(x) \; V(y_1, y_2)$$

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$$S(\boxed{J_1} \; \boxed{P_2} \; \boxed{M_3} \; \boxed{z_4} \; \boxed{h_5} \; \boxed{\ell_6})$$

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$$S(\boxed{x} \ \boxed{y_1} \ \text{zag} \ \boxed{y_2}) \rightarrow N(\boxed{x}) \ V(\boxed{y_1}, \ \boxed{y_2})$$

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$$S(\boxed{J_1} \ \boxed{P_2} \ \boxed{M_3} \ \boxed{z_4} \ \boxed{h_5} \ \boxed{\ell_6})$$
$$N(\boxed{J_1}) \quad \quad \quad V(\boxed{P_2} \ \boxed{M_3}, \ \boxed{h_5} \ \boxed{\ell_6})$$

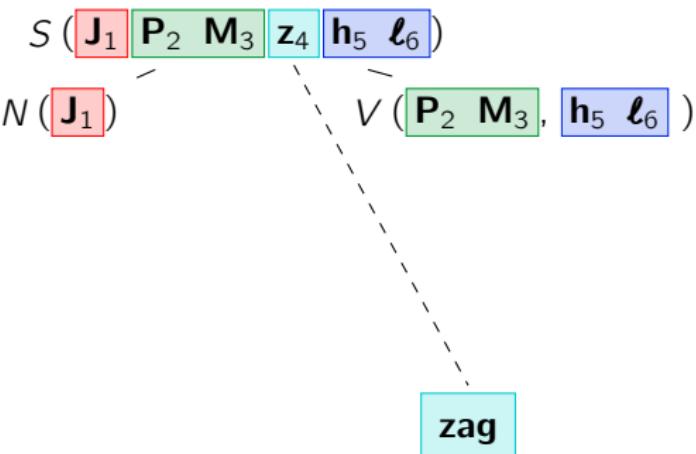
lexicalized LCFRS G :

$$S(\boxed{x} \ \boxed{y_1} \ \boxed{\text{zag}} \ \boxed{y_2}) \rightarrow N(\boxed{x}) \ V(\boxed{y_1}, \boxed{y_2})$$

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(dat)

zag

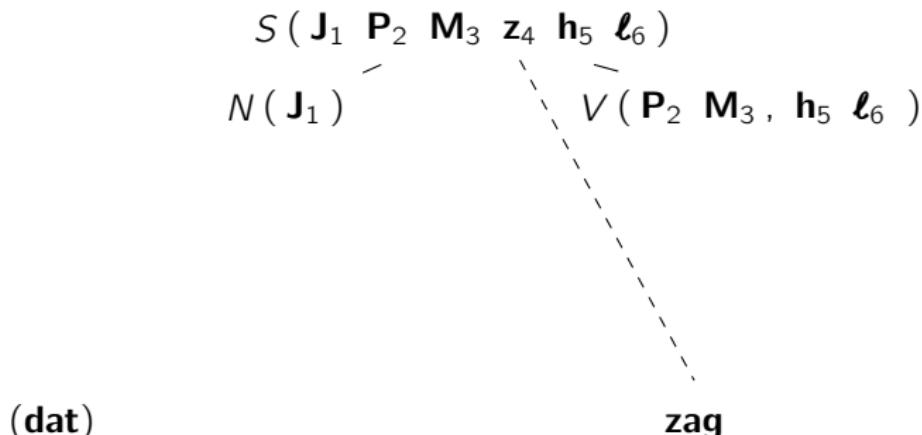
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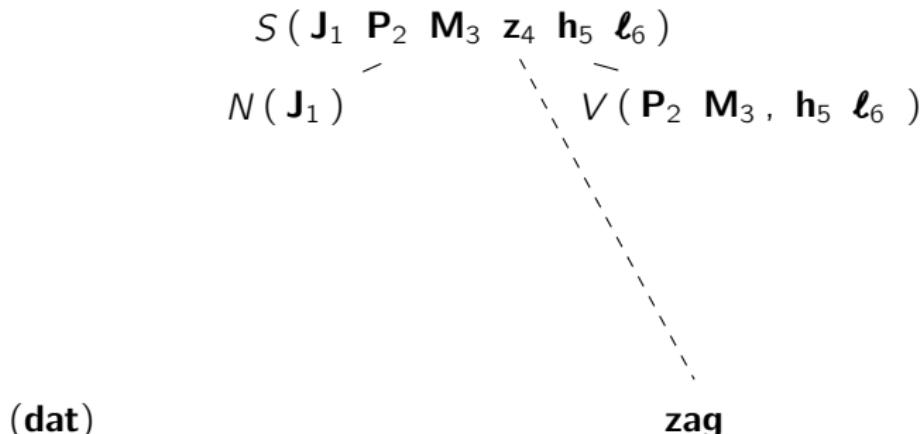
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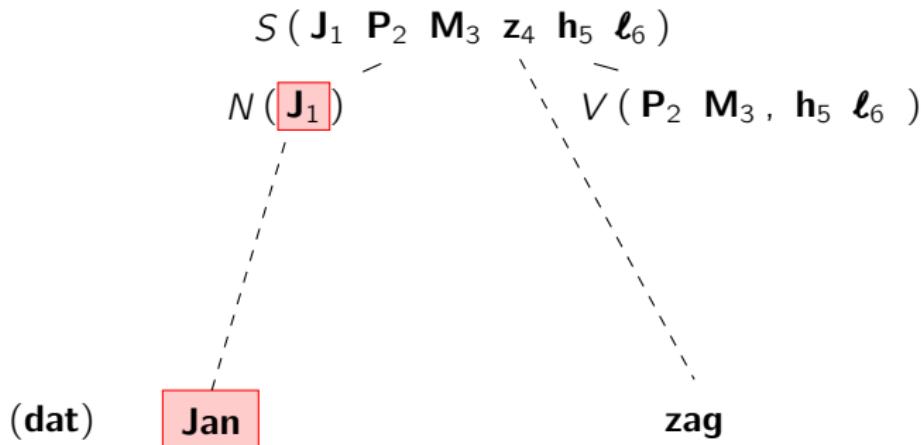
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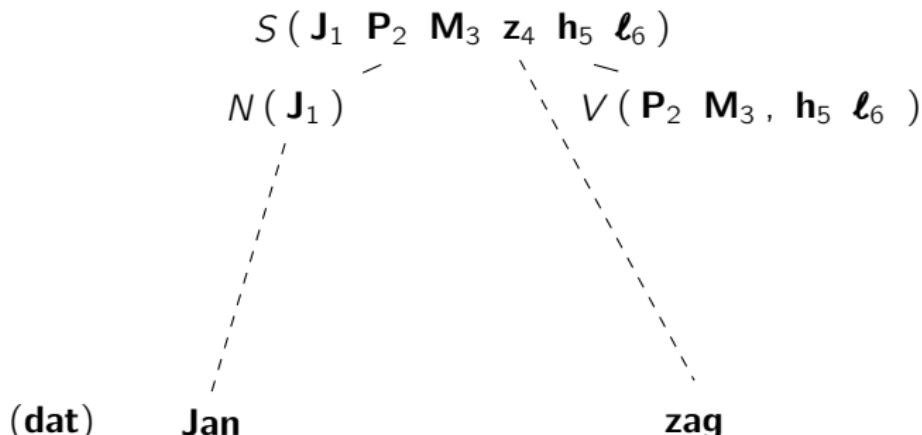
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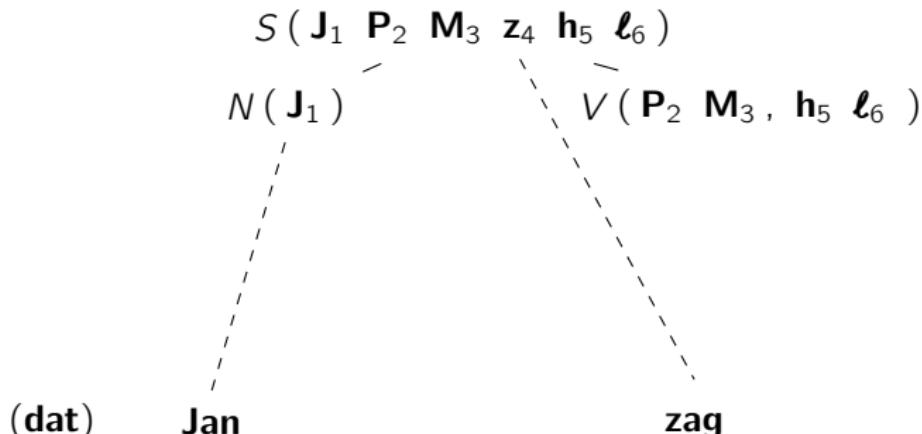
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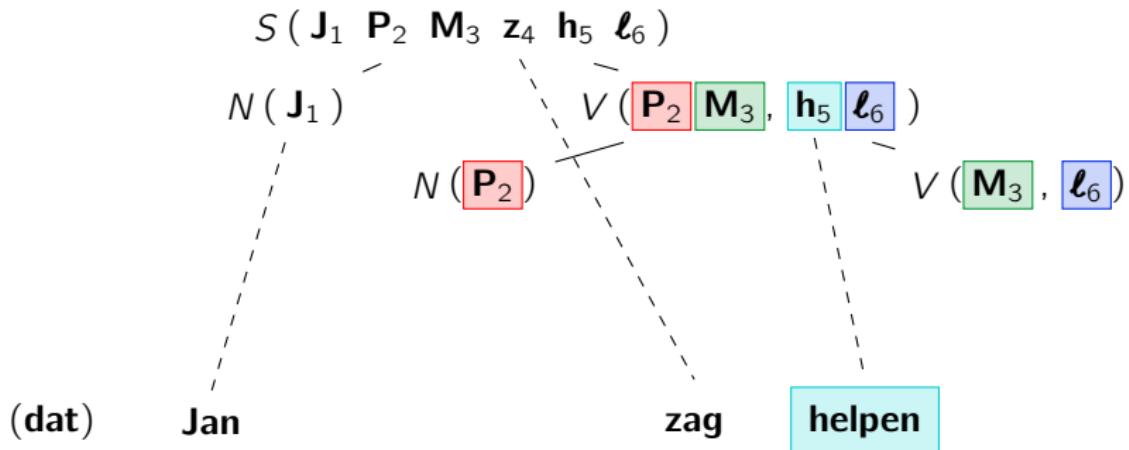
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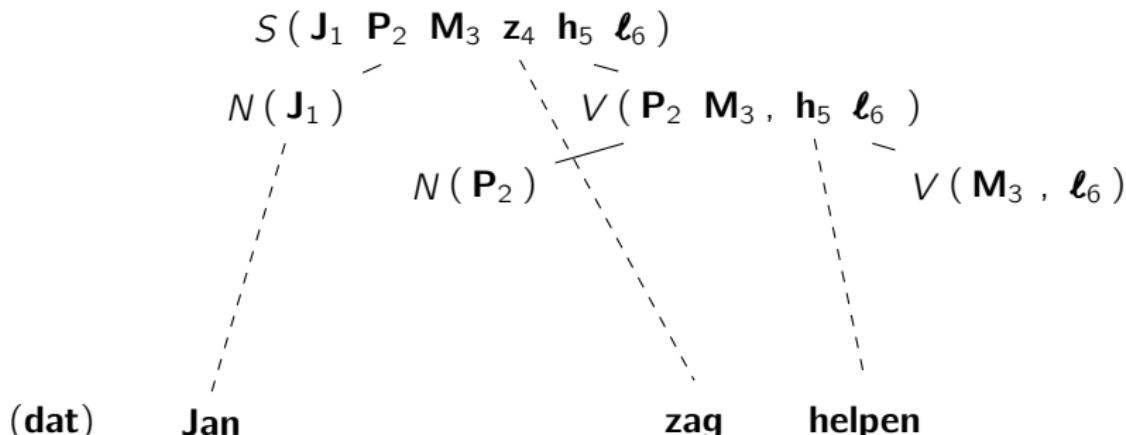
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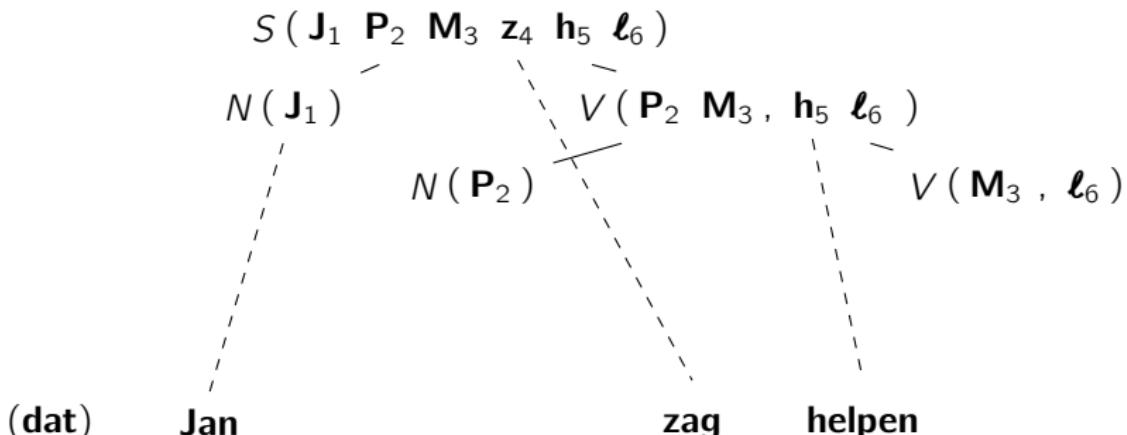
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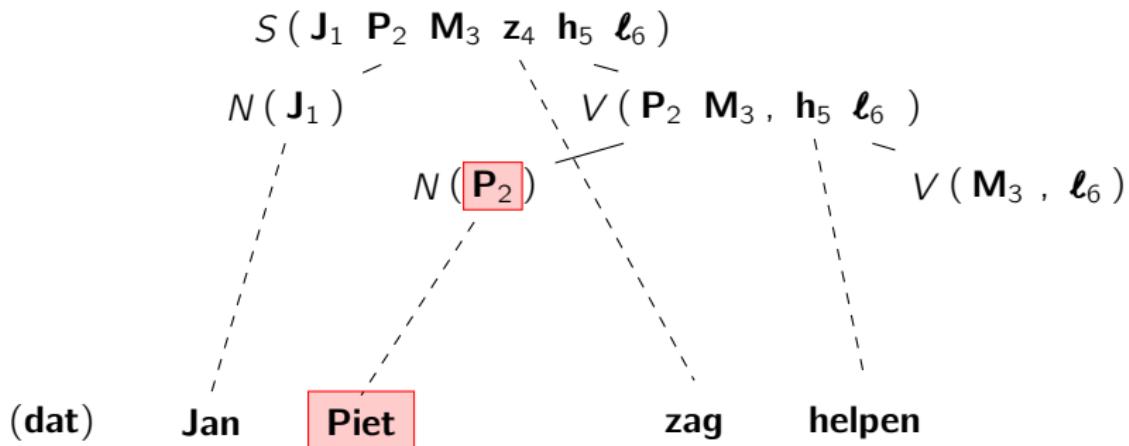
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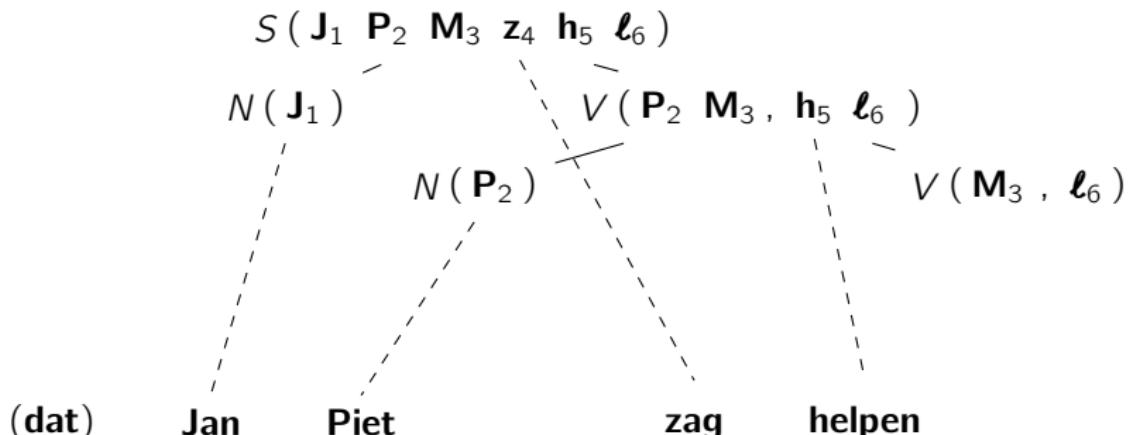
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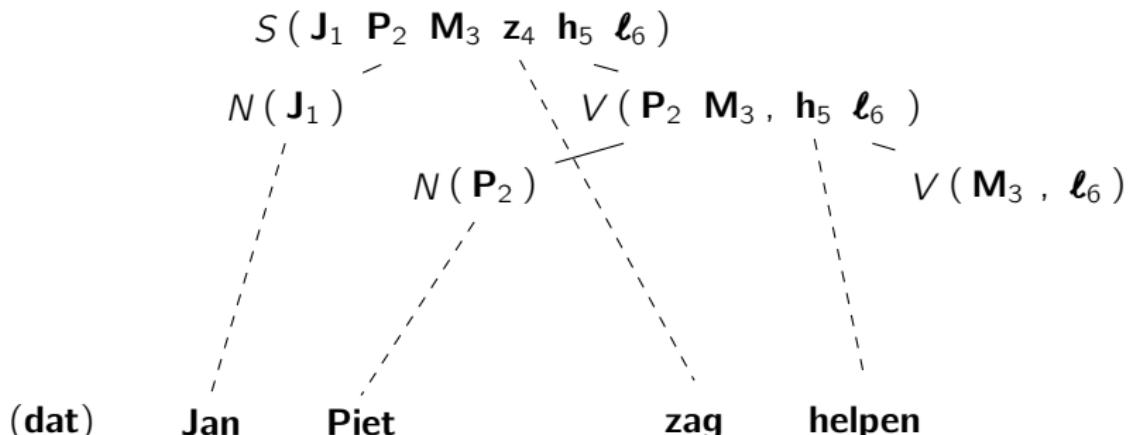
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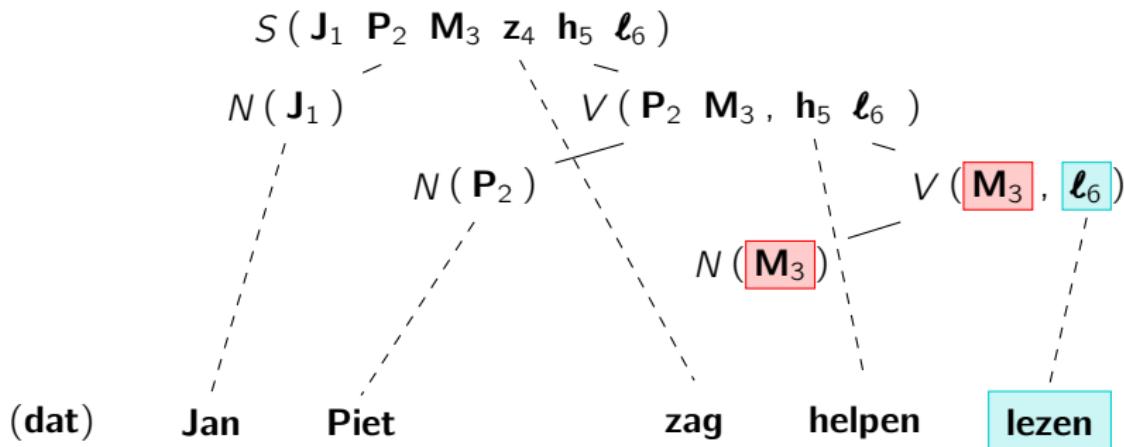
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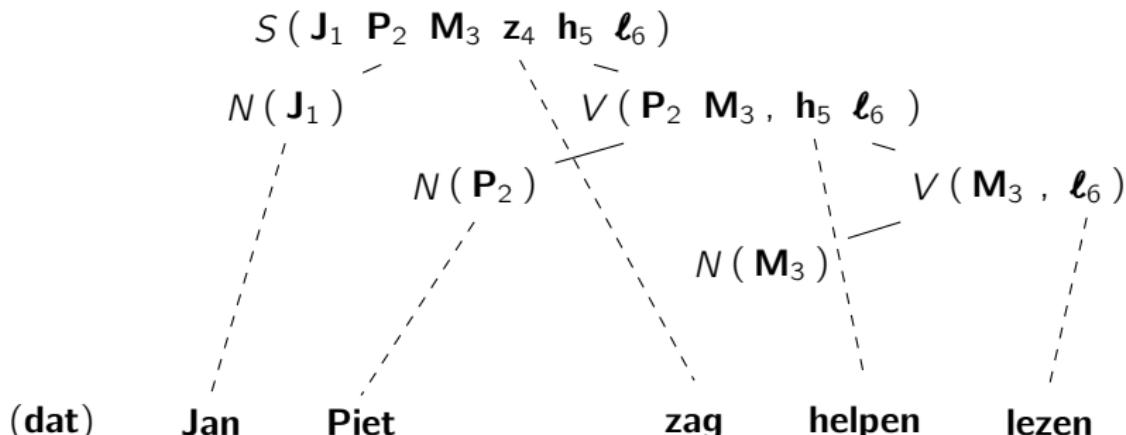
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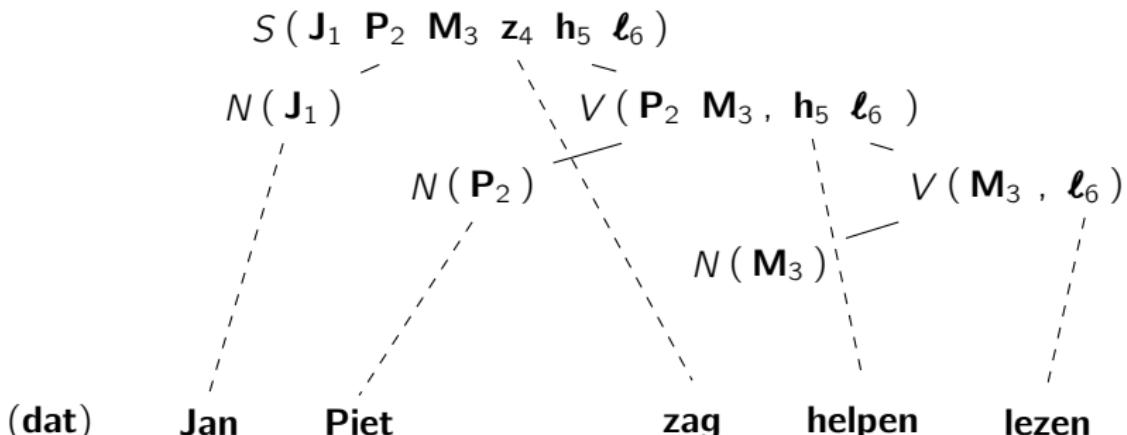
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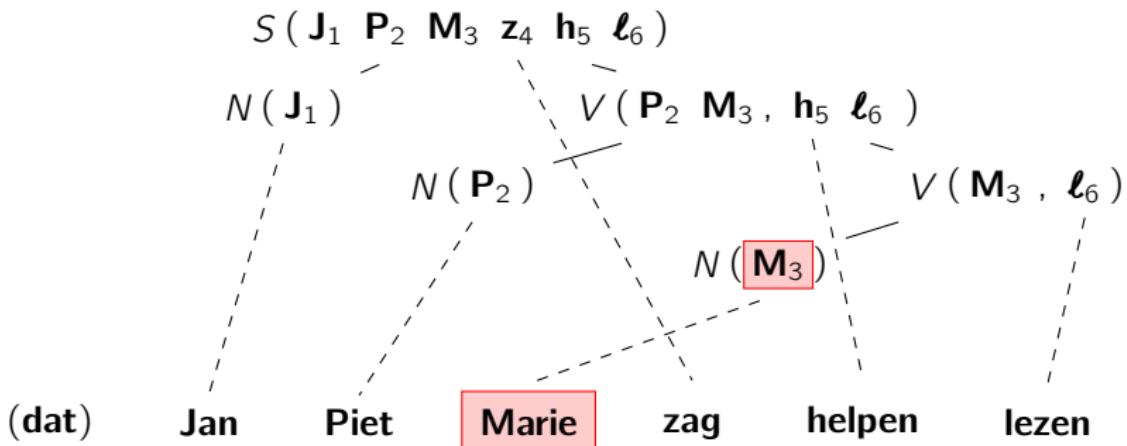
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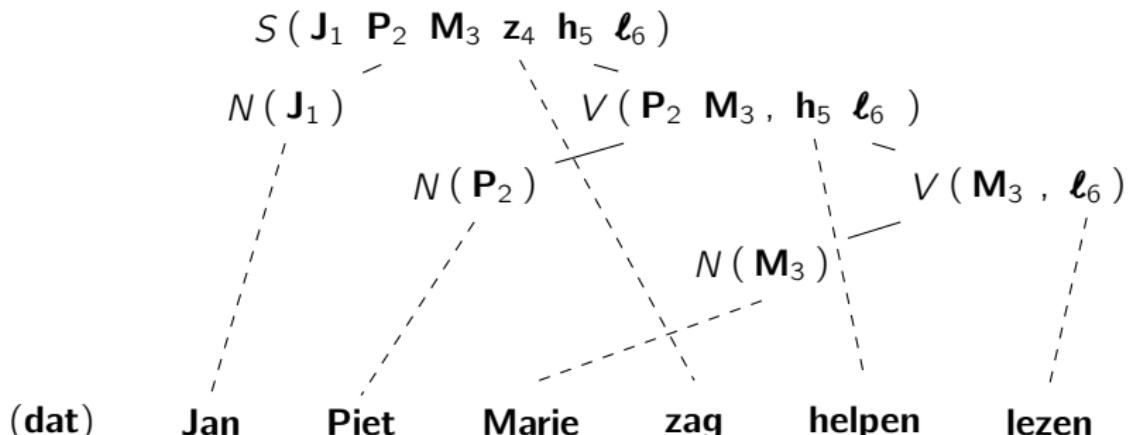
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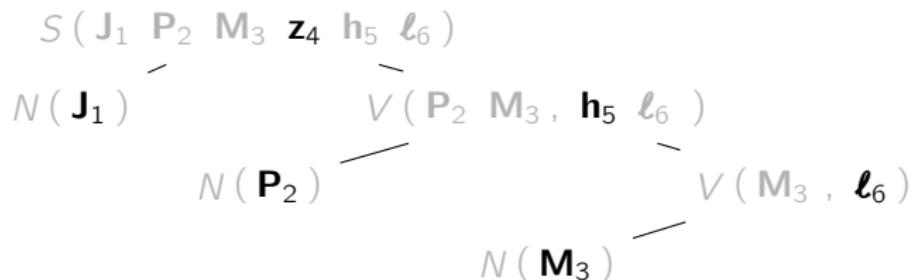
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parsing(**Jan**₁ **Piet**₂ **Marie**₃ **zag**₄ **helpen**₅ **lezen**₆) =



projection to hybrid tree

E : natural language \mathcal{H} : set of dependency trees

parsing : $E \rightarrow \mathcal{H}$

$$\text{parsing}(e) = \text{proj} \left(\underset{\substack{d \in D_G: \\ \text{str}(\text{proj}(d)) = e}}{\operatorname{argmax}} P(d \mid (G, p)) \right)$$

where

- ▶ (G, p) : probabilistic lexicalized LCFRS
 $(D_G$: set of proof trees of G)
- ▶ $\text{proj} : D_G \rightarrow \mathcal{H}$
- ▶ $P(r_1 \dots r_n \mid (G, p)) = p(r_1) \cdot \dots \cdot p(r_n)$

Outline

- ▶ Syntax of natural language sentences
 - ▶ hybrid trees: phrase structure trees, dependency trees
- ▶ Ambiguity and parsing
 - ▶ probabilistic language models
- ▶ A particular “simple and revealing grammar” model
 - ▶ probabilistic LCFRS
- ▶ Probabilistic hybrid grammars

grammar formalisms	phrase structure trees		dependency trees		parsing compl.
	cont.	discont.	proj.	non-proj.	

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LCFRS	x	x	x	x	$\mathcal{O}(n^{3 \cdot \text{fanout}(G)})$ (binar.)	

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trade-off: “richness” of syntactic structures versus parsing complexity

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	-	-	-	-	
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trade-off: “richness” of syntactic structures versus parsing complexity

Idea:

split generation of syntactic structures
from generation of strings

Outline

- ▶ ...
- ▶ Probabilistic hybrid grammars

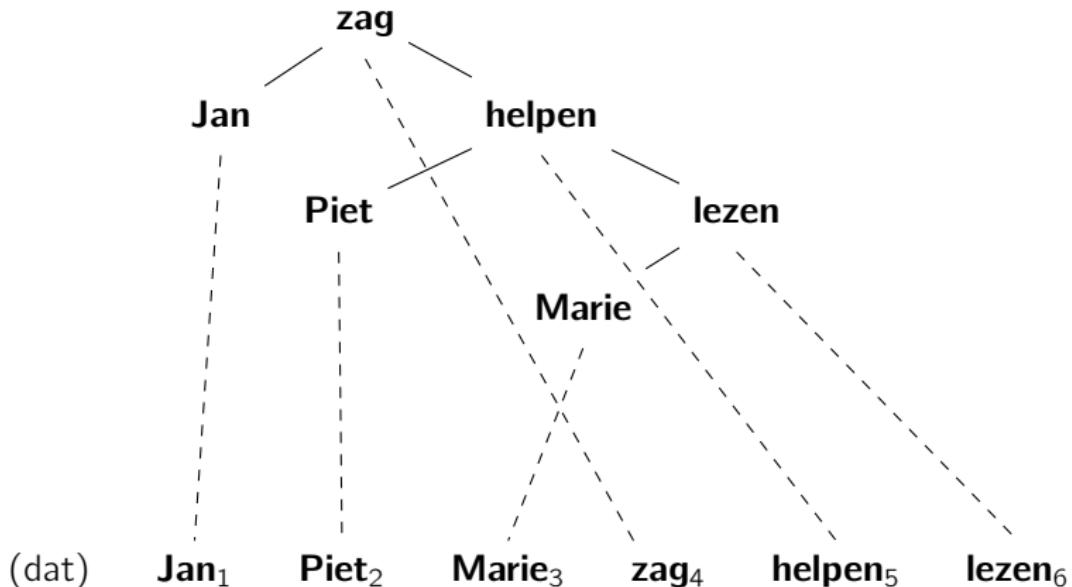
[Nederhof, HV 14]

Hybrid Grammars for Discontinuous Parsing.
COLING 2014

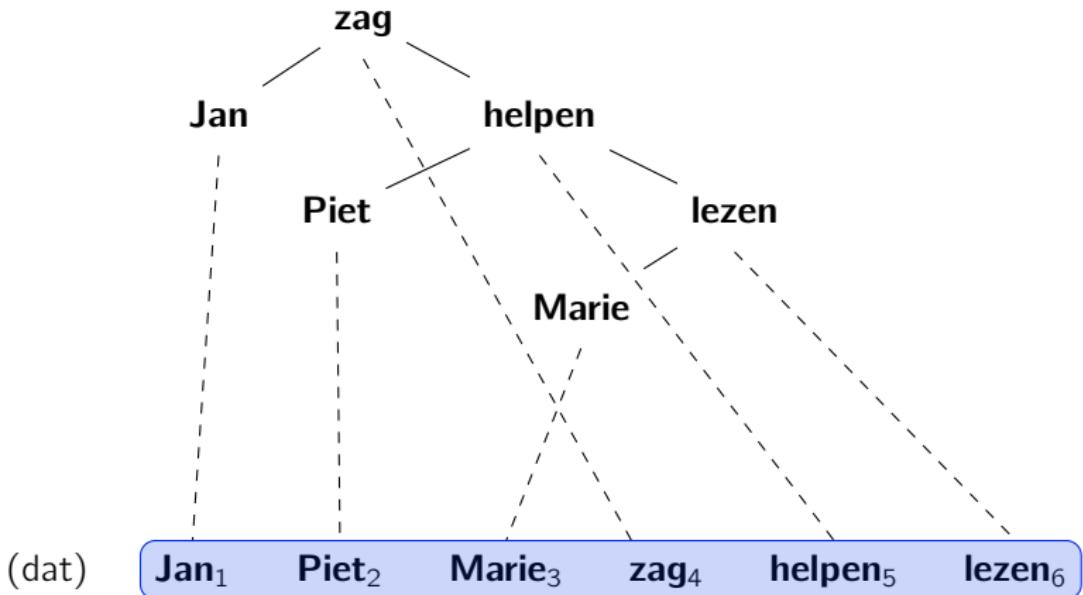
[Gebhardt, Nederhof, HV 17]

Hybrid grammars for parsing of discontinuous phrase structures and
non-projective dependency structures,
accepted for publication 2016, Computational Linguistics.

hybrid tree:

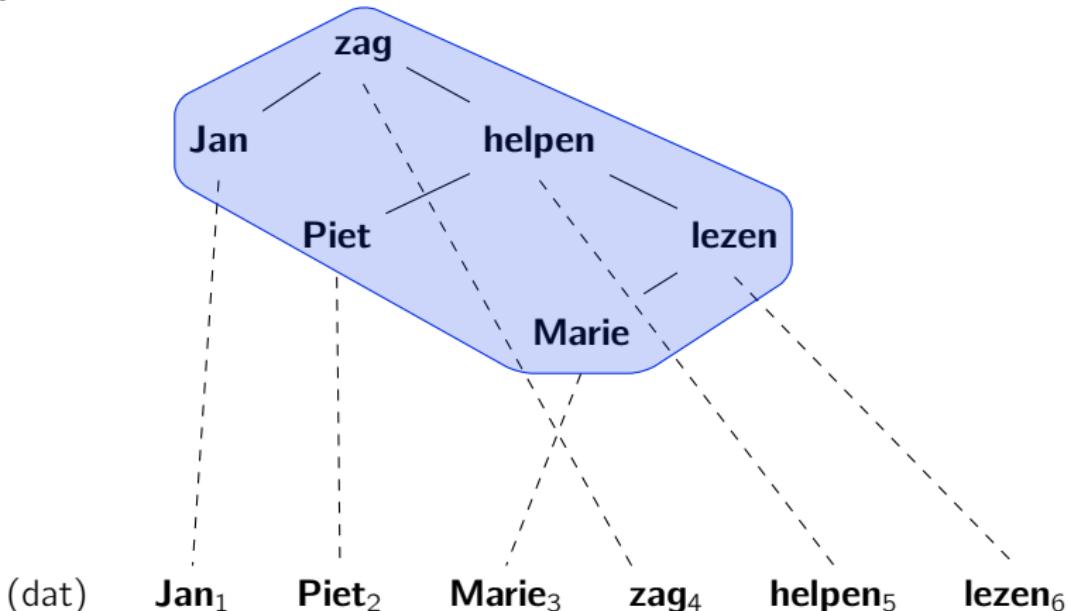


hybrid tree:



string grammar

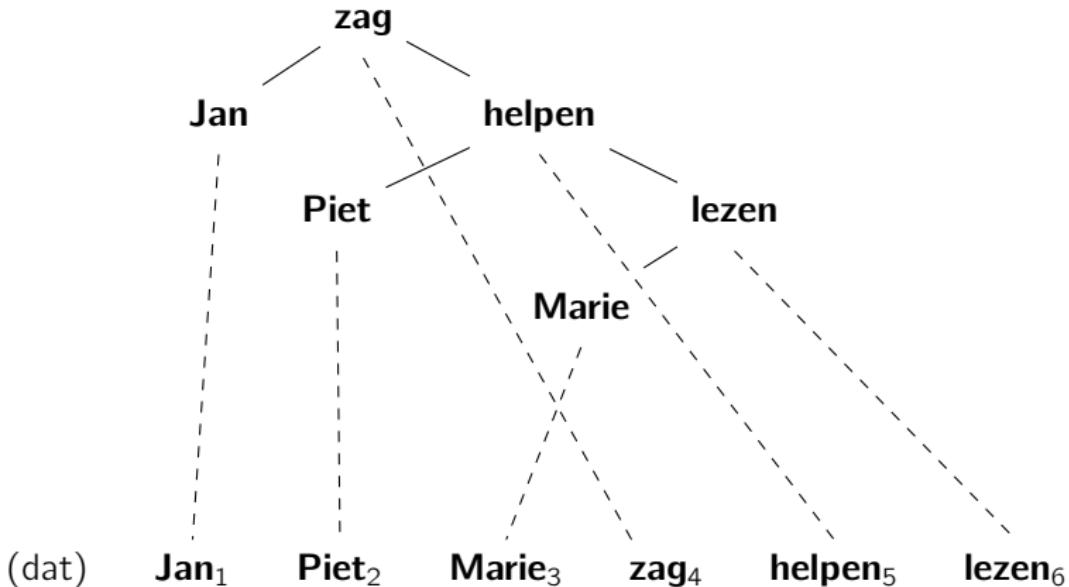
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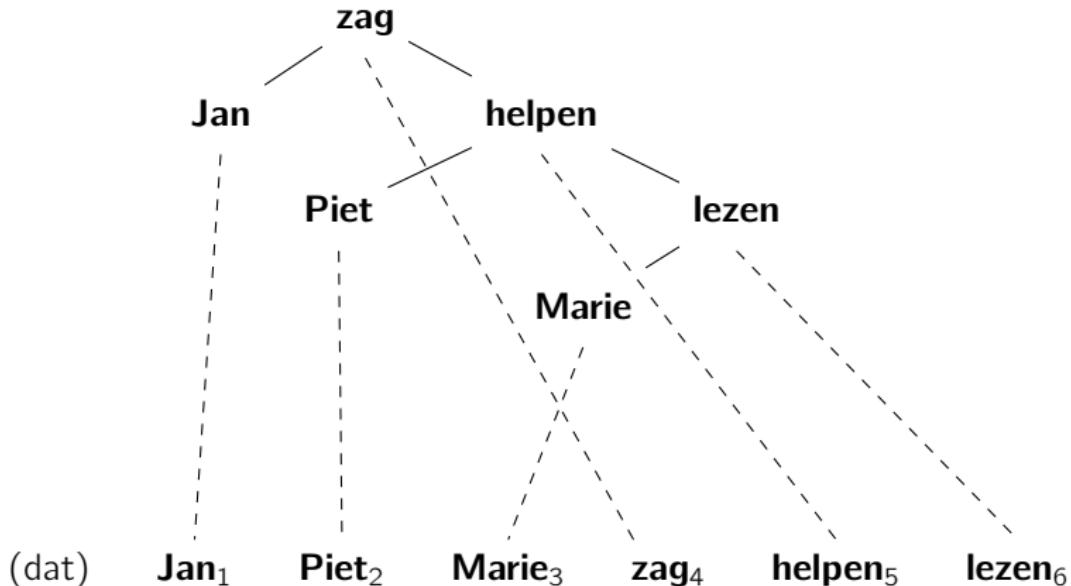
+ tree grammar

hybrid tree:



string grammar + synchronization + tree grammar

hybrid tree:



hybrid grammar = string grammar + synchronization + tree grammar

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string grammar:

- ▶ regular grammars / finite-state automata
- ▶ context-free grammars
- ▶ macro grammars [Fischer 68]
- ▶ linear context-free rewriting systems (LCFRS)

tree grammar:

- ▶ regular tree grammars [Brainerd 69]
- ▶ context-free tree grammars [Rounds 70; Engelfriet, Schmidt 77]
- ▶ simple definite clause programs (sDCP)
[Deransart, Maluszynski 85]

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(LCFRS,sDCP)-hybrid grammars \rightsquigarrow hybrid grammars

Outline

- ▶ ...
- ▶ Probabilistic hybrid grammars
 - ▶ simple definite clause programs (sDCP)
 - ▶ (LCFRS,sDCP)-hybrid grammars
 - ▶ parsing with hybrid grammars
 - ▶ how to obtain a hybrid grammar for E ?
 - ▶ experiments

simple definite clause programs (sDCPs):

≈ attribute grammars

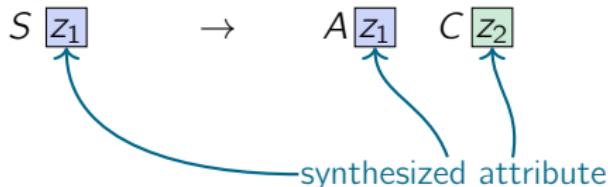
[Knuth 68]

$S \rightarrow A C$

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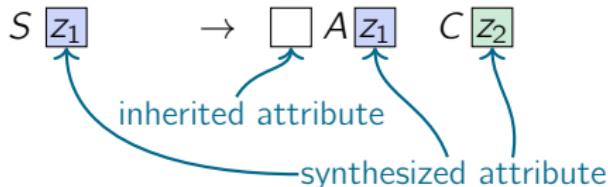
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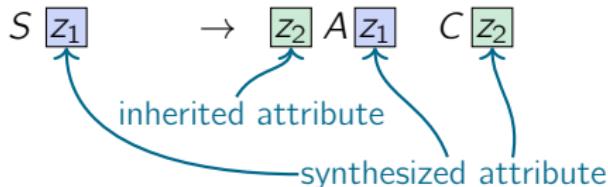
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$$[z_1] A \boxed{h \\ z_2 \\ z_1} \rightarrow B [z_2]$$

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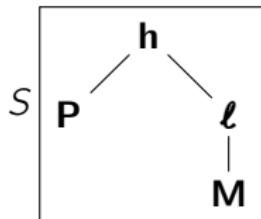
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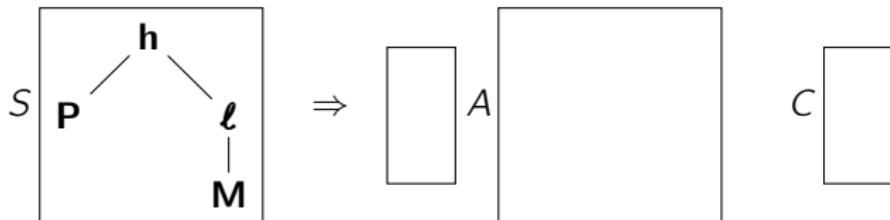
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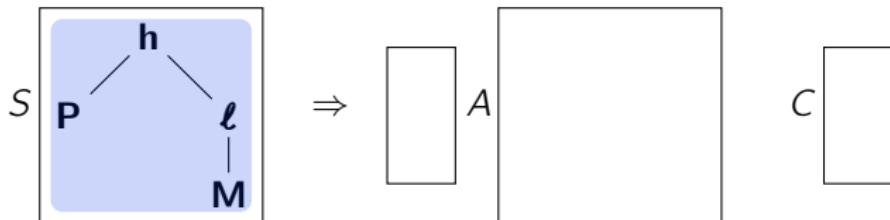
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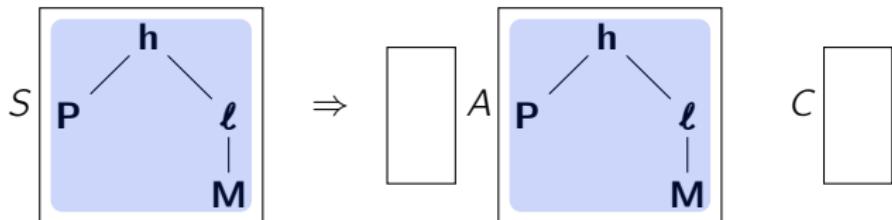
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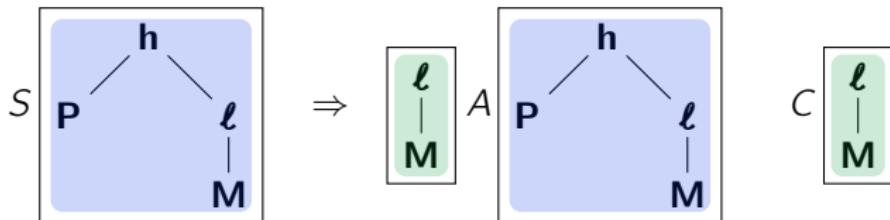
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$$B \blacksquare P \rightarrow \varepsilon$$

$$C \begin{array}{c} \ell \\ | \\ z_1 \end{array} \rightarrow D [z_1]$$
$$D \blacksquare M \rightarrow \varepsilon$$



simple definite clause programs (sDCPs):

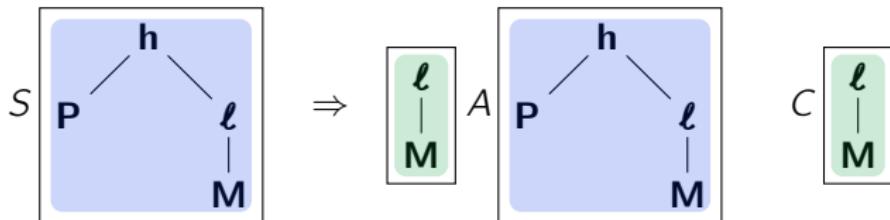
≈ attribute grammars

[Knuth 68]

$$S [z_1] \rightarrow [z_2] A [z_1] C [z_2]$$

$$[z_1] A \boxed{h} \begin{array}{c} z_2 \\ z_1 \end{array} \rightarrow B [z_2] \quad B \boxed{P} \rightarrow \varepsilon$$

$$C \boxed{\ell} \begin{array}{c} z_1 \end{array} \rightarrow D [z_1] \quad D \boxed{M} \rightarrow \varepsilon$$



simple definite clause programs (sDCPs):

≈ attribute grammars

[Knuth 68]

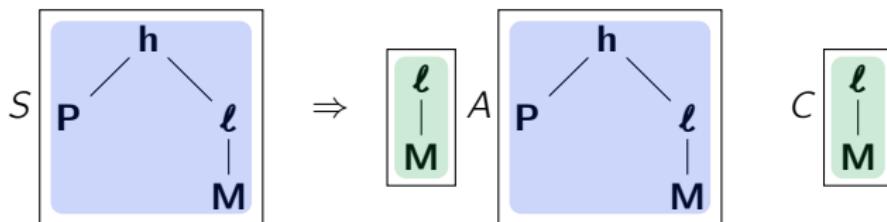
$$S [z_1] \rightarrow [z_2] A [z_1] C [z_2]$$

$$[z_1] A \begin{array}{c} h \\ / \quad \backslash \\ z_2 \quad z_1 \end{array} \rightarrow B [z_2]$$

$$B \text{ P} \rightarrow \varepsilon$$

$$C \begin{array}{c} \ell \\ | \\ z_1 \end{array} \rightarrow D [z_1]$$

$$D \text{ M} \rightarrow \varepsilon$$



simple definite clause programs (sDCPs):

≈ attribute grammars

[Knuth 68]

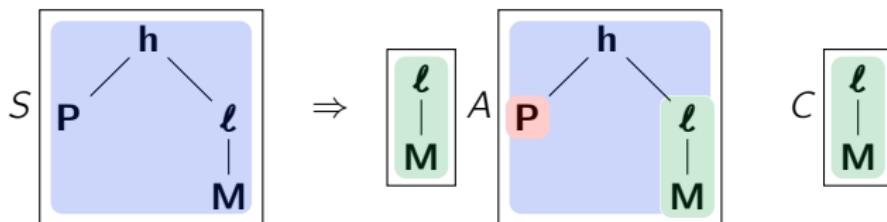
$$S [z_1] \rightarrow [z_2] A [z_1] C [z_2]$$

$$[z_1] A \begin{array}{c} h \\ / \quad \backslash \\ z_2 \quad z_1 \end{array} \rightarrow B [z_2]$$

$$B \text{ P} \rightarrow \varepsilon$$

$$C \begin{array}{c} \ell \\ | \\ z_1 \end{array} \rightarrow D [z_1]$$

$$D \text{ M} \rightarrow \varepsilon$$



simple definite clause programs (sDCPs):

≈ attribute grammars

[Knuth 68]

$$S [z_1] \rightarrow [z_2] A [z_1] C [z_2]$$

$$[z_1] A \begin{array}{c} h \\ / \quad \backslash \\ z_2 \quad z_1 \end{array} \rightarrow B [z_2]$$

$$B \blacksquare P \rightarrow \varepsilon$$

$$C \begin{array}{c} \ell \\ | \\ z_1 \end{array} \rightarrow D [z_1]$$

$$D \blacksquare M \rightarrow \varepsilon$$

$$S \begin{array}{c} h \\ / \quad \backslash \\ P \quad \ell \\ | \\ \ell \quad M \end{array} \Rightarrow$$

$$\ell \mid M$$

$$A \begin{array}{c} h \\ / \quad \backslash \\ P \quad \ell \\ | \\ \ell \quad M \end{array}$$

$$\ell \mid M$$

$$\Rightarrow B \blacksquare P$$

$$\ell \mid M$$

simple definite clause programs (sDCPs):

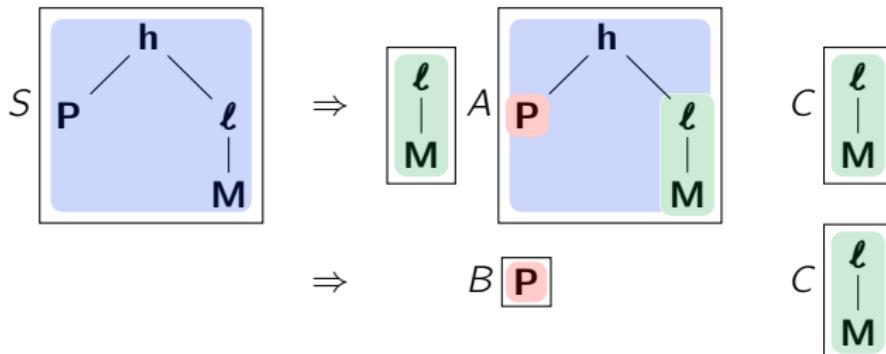
≈ attribute grammars

[Knuth 68]

$$S [z_1] \rightarrow [z_2] A [z_1] C [z_2]$$

$$[z_1] A \boxed{h} \begin{array}{c} z_2 \\ z_1 \end{array} \rightarrow B [z_2] \quad B \boxed{P} \rightarrow \varepsilon$$

$$C \boxed{\ell} \begin{array}{c} z_1 \end{array} \rightarrow D [z_1] \quad D \boxed{M} \rightarrow \varepsilon$$



simple definite clause programs (sDCPs):

≈ attribute grammars

[Knuth 68]

$$S [z_1] \rightarrow [z_2] A [z_1] C [z_2]$$

$$[z_1] A \begin{array}{c} h \\ / \quad \backslash \\ z_2 \quad z_1 \end{array} \rightarrow B [z_2]$$
$$B \blacksquare P \rightarrow \varepsilon$$

$$C \begin{array}{c} \ell \\ | \\ z_1 \end{array} \rightarrow D [z_1]$$
$$D \blacksquare M \rightarrow \varepsilon$$

$$S \begin{array}{c} h \\ / \quad \backslash \\ P \quad \ell \\ \quad \quad | \\ \quad \quad M \end{array} \Rightarrow^2 B \blacksquare P$$

$$C \begin{array}{c} \ell \\ | \\ M \end{array}$$

simple definite clause programs (sDCPs):

≈ attribute grammars

[Knuth 68]

$$S [z_1] \rightarrow [z_2] A [z_1] C [z_2]$$

$$[z_1] A \begin{array}{c} h \\ / \quad \backslash \\ z_2 \quad z_1 \end{array} \rightarrow B [z_2]$$

$$B \boxed{P} \rightarrow \epsilon$$

$$C \begin{array}{c} \ell \\ | \\ z_1 \end{array} \rightarrow D [z_1]$$

$$D \boxed{M} \rightarrow \epsilon$$

$$S \boxed{P} \begin{array}{c} h \\ / \quad \backslash \\ \ell \quad M \end{array} \Rightarrow^2 B \boxed{P}$$

$$C \boxed{\ell \mid M}$$

simple definite clause programs (sDCPs):

≈ attribute grammars

[Knuth 68]

$$S [z_1] \rightarrow [z_2] A [z_1] C [z_2]$$

$$[z_1] A \begin{array}{c} h \\ / \quad \backslash \\ z_2 \quad z_1 \end{array} \rightarrow B [z_2]$$
$$B \blacksquare P \rightarrow \epsilon$$

$$C \begin{array}{c} \ell \\ | \\ z_1 \end{array} \rightarrow D [z_1]$$
$$D \blacksquare M \rightarrow \epsilon$$

$$S \begin{array}{c} h \\ / \quad \backslash \\ P \quad \ell \\ \ell \quad \mid \quad M \end{array} \Rightarrow^3$$

$$C \begin{array}{c} \ell \\ | \\ M \end{array}$$

simple definite clause programs (sDCPs):

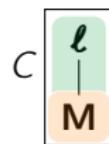
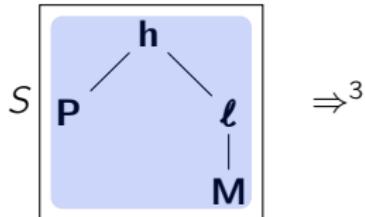
≈ attribute grammars

[Knuth 68]

$$S [z_1] \rightarrow [z_2] A [z_1] C [z_2]$$

$$[z_1] A \begin{array}{c} h \\ / \quad \backslash \\ z_2 \quad z_1 \end{array} \rightarrow B [z_2]$$
$$B \blacksquare P \rightarrow \epsilon$$

$$C \begin{array}{c} \ell \\ | \\ z_1 \end{array} \rightarrow D [z_1]$$
$$D \blacksquare M \rightarrow \epsilon$$



simple definite clause programs (sDCPs):

≈ attribute grammars

[Knuth 68]

$$S [z_1] \rightarrow [z_2] A [z_1] C [z_2]$$

$$[z_1] A \boxed{h} \begin{array}{c} z_2 \\ z_1 \end{array} \rightarrow B [z_2] \quad B \boxed{P} \rightarrow \varepsilon$$

$$C \boxed{\ell} \begin{array}{c} z_1 \end{array} \rightarrow D [z_1] \quad D \boxed{M} \rightarrow \varepsilon$$

$$S \boxed{P} \boxed{h} \begin{array}{c} \ell \\ M \end{array} \Rightarrow^3 \begin{array}{c} \Rightarrow \\ \Rightarrow \end{array}$$

$$C \boxed{\ell} \begin{array}{c} M \end{array} \Rightarrow D \boxed{M}$$

simple definite clause programs (sDCPs):

≈ attribute grammars

[Knuth 68]

$$S [z_1] \rightarrow [z_2] A [z_1] C [z_2]$$

$$[z_1] A \begin{array}{c} h \\ / \quad \backslash \\ z_2 \quad z_1 \end{array} \rightarrow B [z_2]$$
$$B \text{ } \blacksquare P \rightarrow \epsilon$$

$$C \begin{array}{c} \ell \\ | \\ z_1 \end{array} \rightarrow D [z_1]$$
$$D \text{ } \blacksquare M \rightarrow \epsilon$$

$$S \begin{array}{c} h \\ / \quad \backslash \\ \blacksquare P \quad \ell \\ \ell \quad \mid \\ \quad \quad M \end{array} \Rightarrow^4 D \text{ } \blacksquare M$$

simple definite clause programs (sDCPs):

≈ attribute grammars

[Knuth 68]

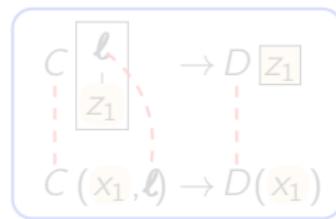
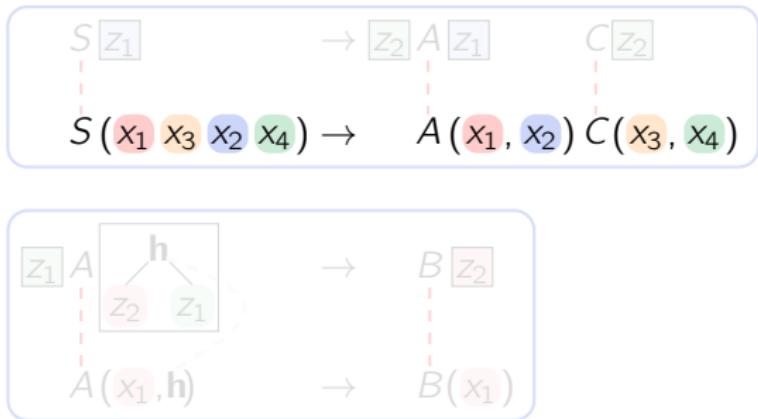
$$S [z_1] \rightarrow [z_2] A [z_1] C [z_2]$$

$$[z_1] A \begin{array}{c} h \\ / \quad \backslash \\ z_2 \quad z_1 \end{array} \rightarrow B [z_2]$$
$$B \blacksquare P \rightarrow \epsilon$$

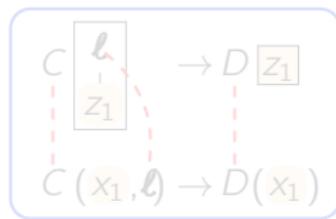
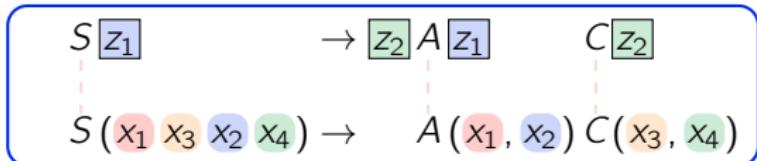
$$C \begin{array}{c} \ell \\ | \\ z_1 \end{array} \rightarrow D [z_1]$$
$$D \blacksquare M \rightarrow \epsilon$$

$$S \begin{array}{c} h \\ / \quad \backslash \\ P \quad \ell \\ \ell \quad \mid \\ \quad \quad M \end{array} \Rightarrow^4 D \blacksquare M$$
$$\Rightarrow \epsilon$$

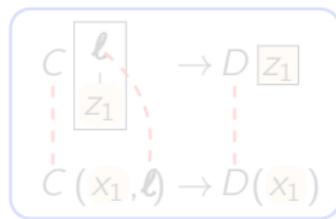
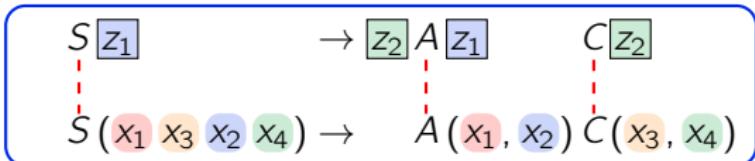
(LCFRS,sDCP)-hybrid grammar:



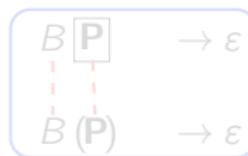
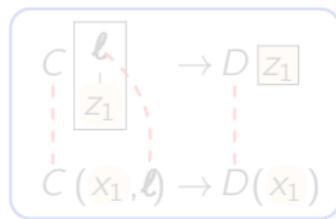
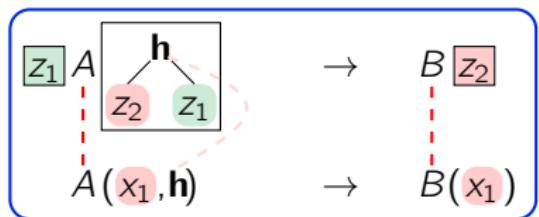
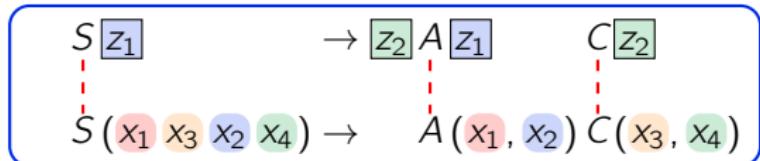
(LCFRS,sDCP)-hybrid grammar:



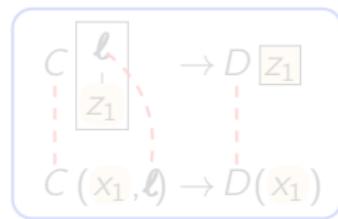
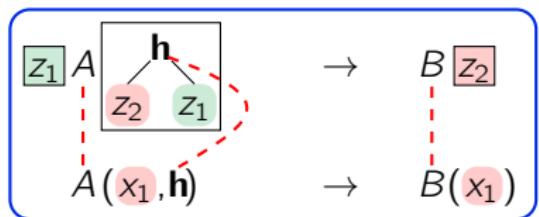
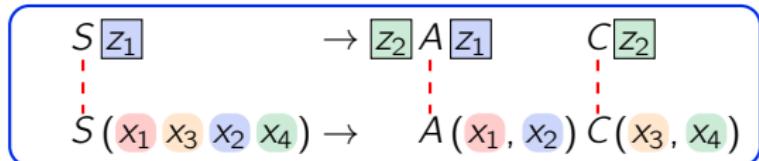
(LCFRS,sDCP)-hybrid grammar:



(LCFRS,sDCP)-hybrid grammar:



(LCFRS,sDCP)-hybrid grammar:



(LCFRS,sDCP)-hybrid grammar:

$$\begin{array}{c}
 S[z_1] \rightarrow [z_2] A[z_1] C[z_2] \\
 S(x_1 \ x_3 \ x_2 \ x_4) \rightarrow A(x_1, x_2) C(x_3, x_4)
 \end{array}$$

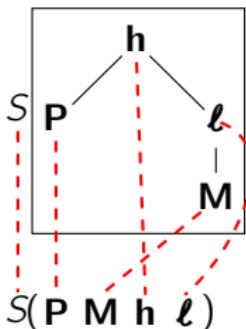
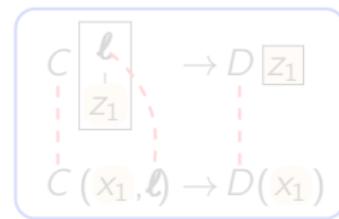
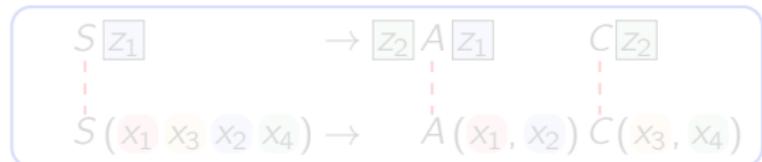
$$\begin{array}{l}
 C[\ell] \rightarrow D[z_1] \\
 C(x_1, \ell) \rightarrow D(x_1)
 \end{array}$$

$$\begin{array}{l}
 B[P] \rightarrow \epsilon \\
 B(P) \rightarrow \epsilon
 \end{array}$$

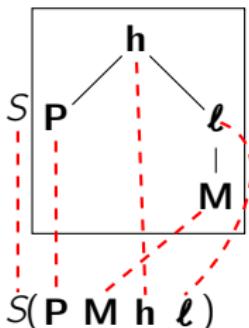
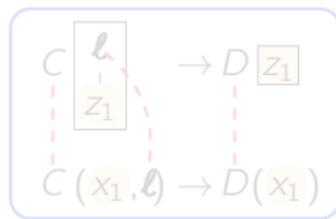
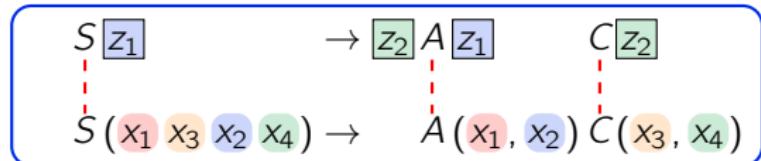
$$\begin{array}{l}
 D[M] \rightarrow \epsilon \\
 D(M) \rightarrow \epsilon
 \end{array}$$

$$\begin{array}{c}
 z_1 A \xrightarrow{\quad} B[z_2] \\
 A(x_1, h) \xrightarrow{\quad} B(x_1)
 \end{array}$$

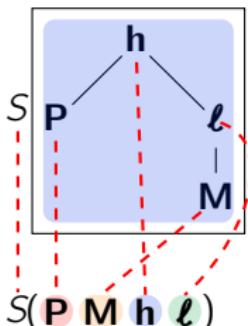
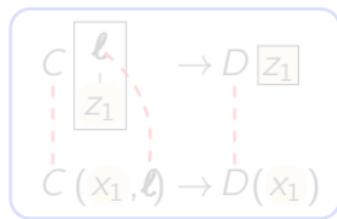
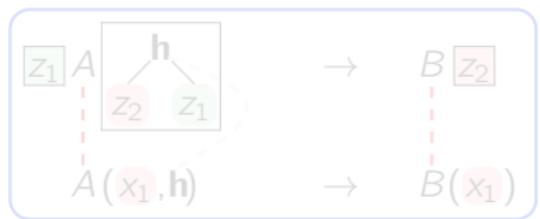
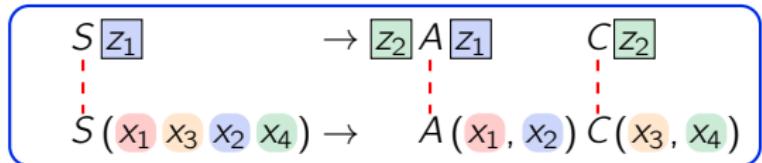
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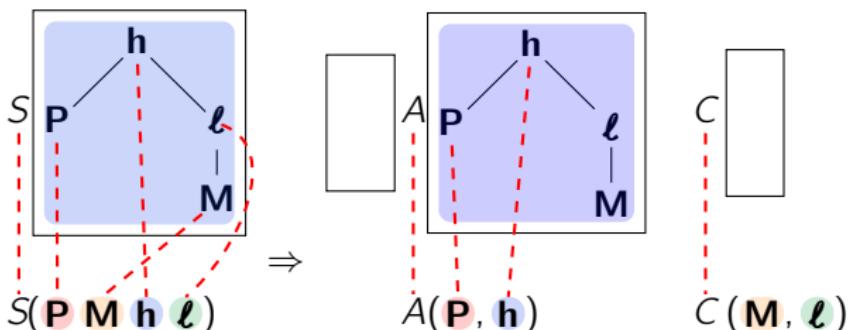
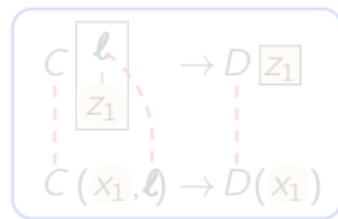
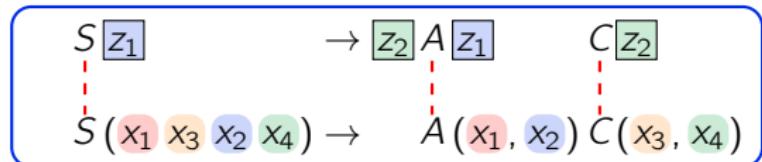
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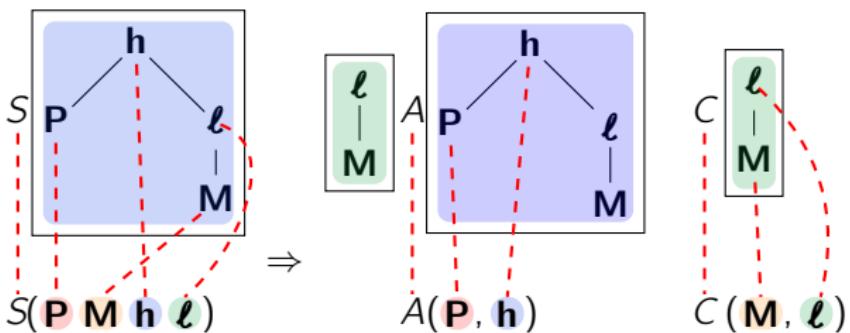
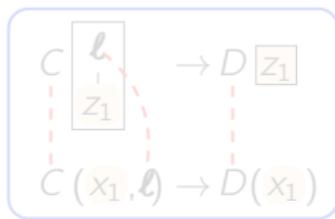
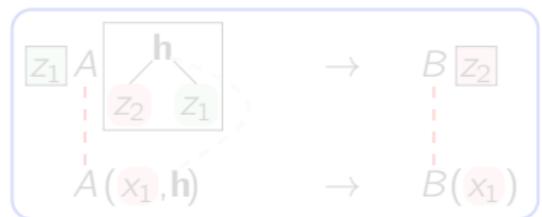
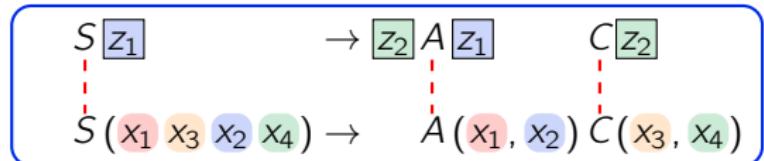
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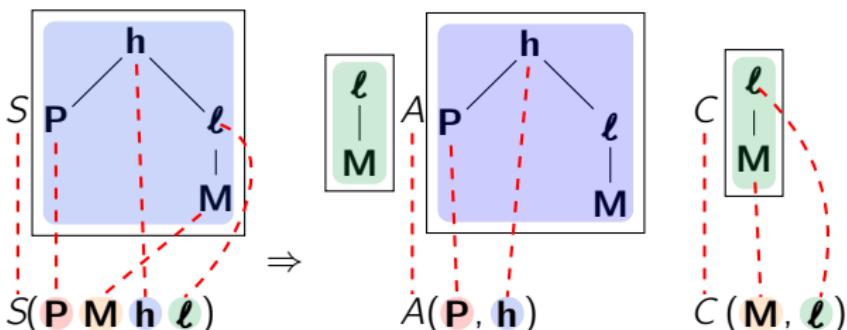
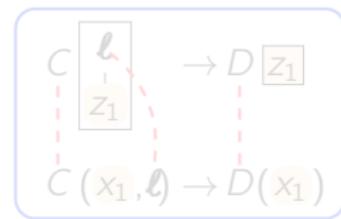
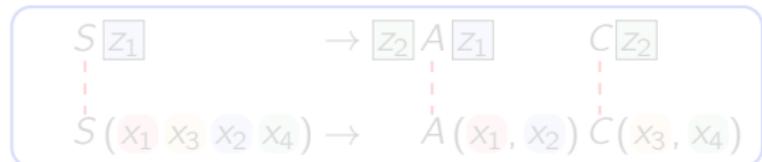
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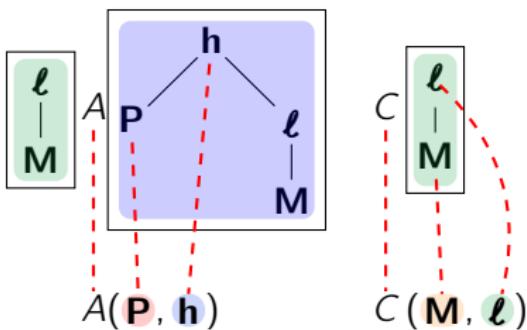
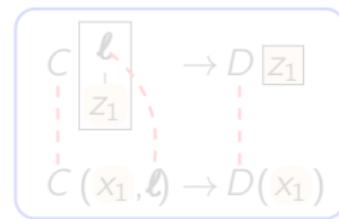
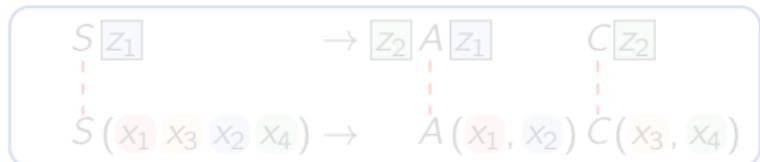
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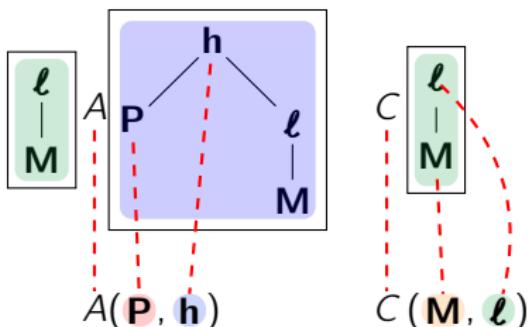
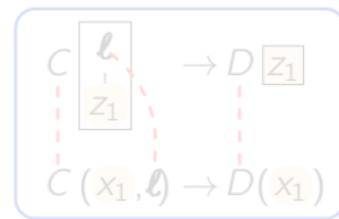
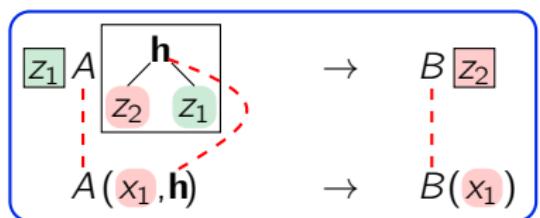
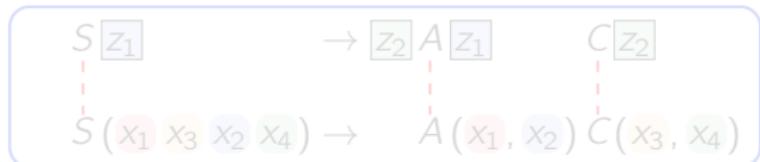
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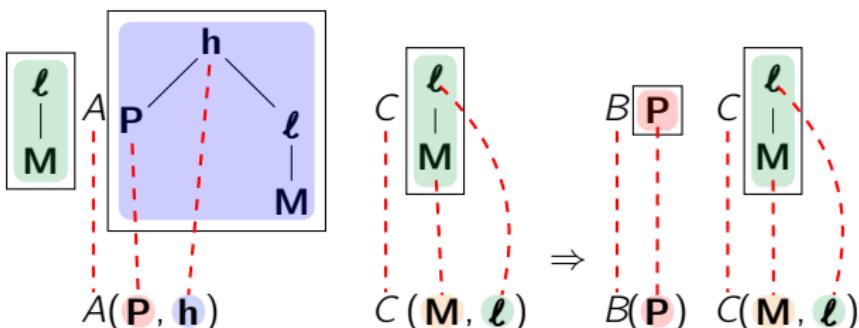
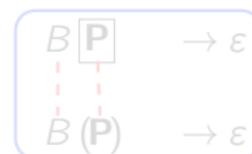
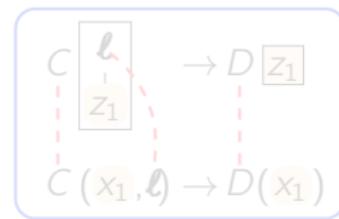
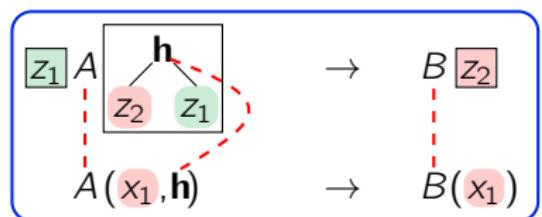
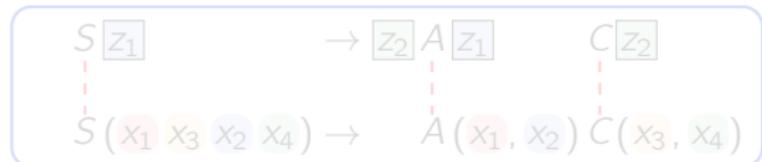
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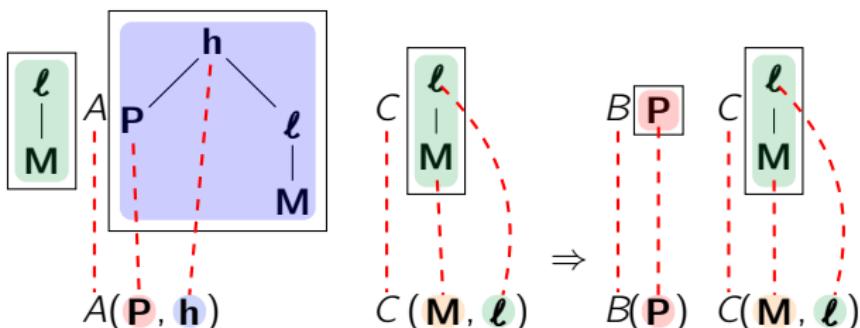
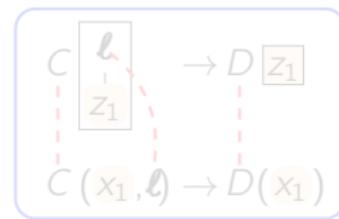
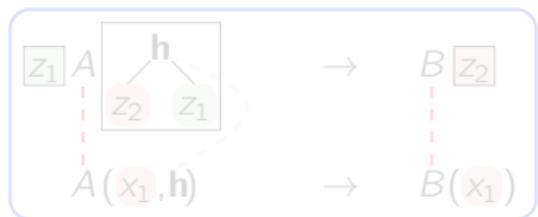
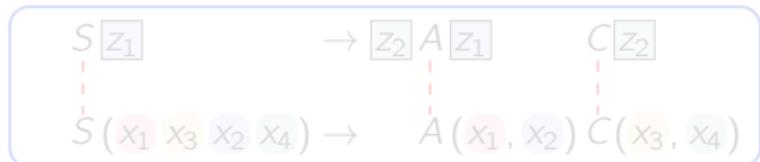
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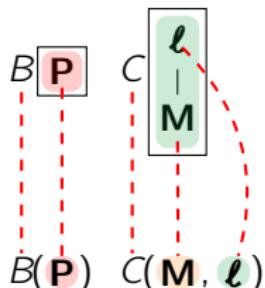
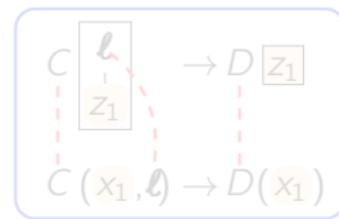
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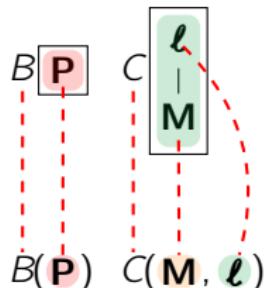
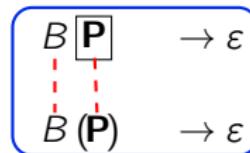
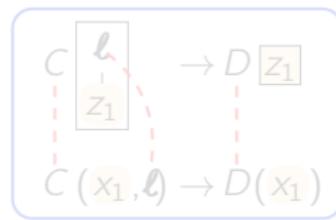
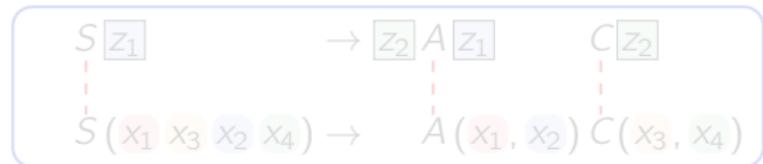
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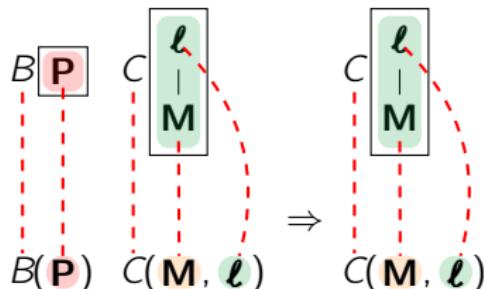
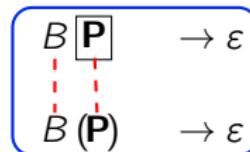
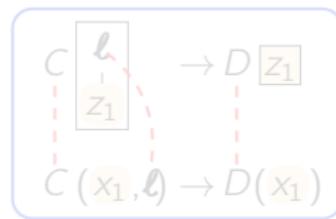
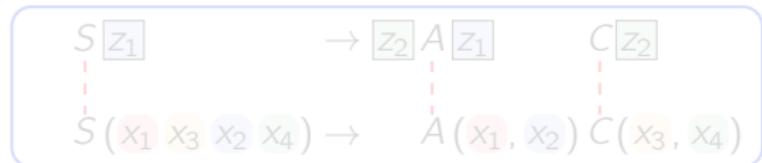
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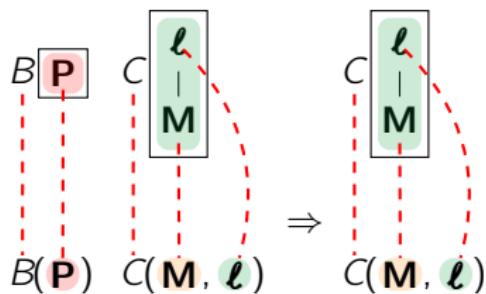
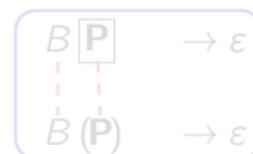
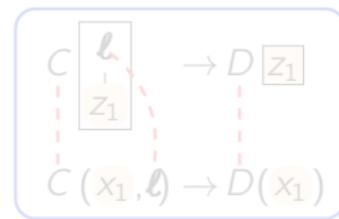
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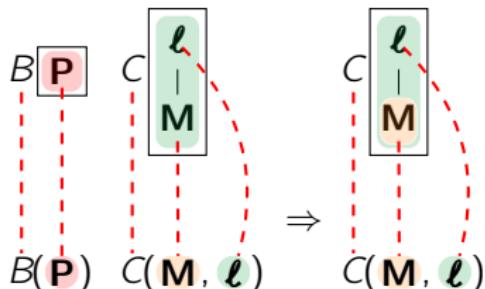
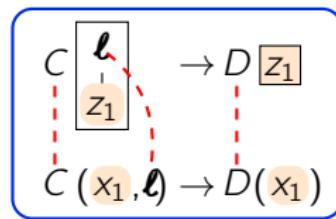
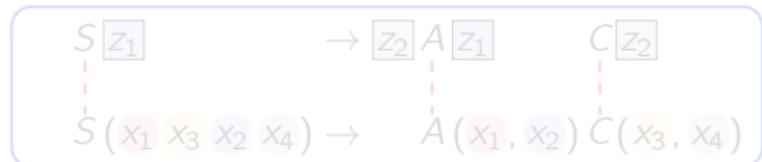
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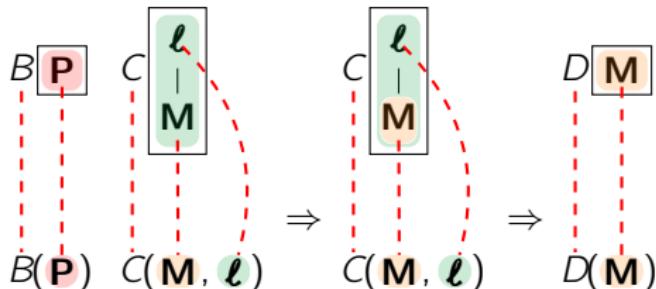
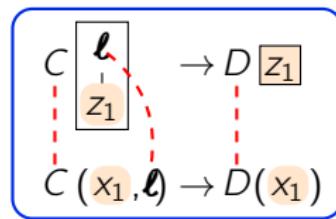
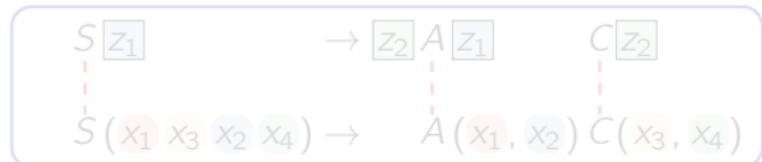
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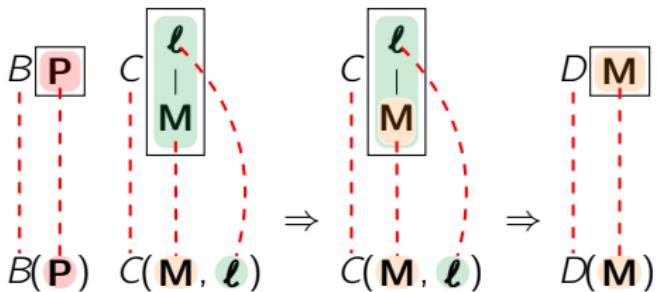
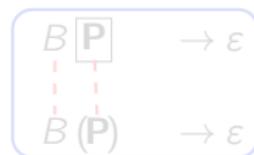
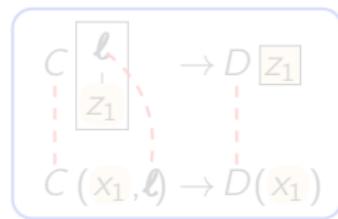
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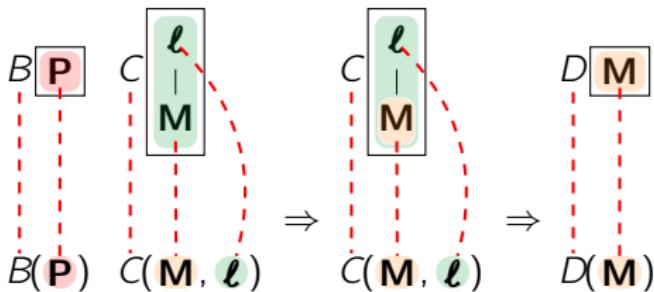
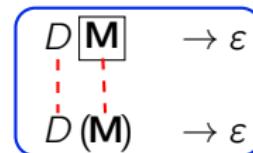
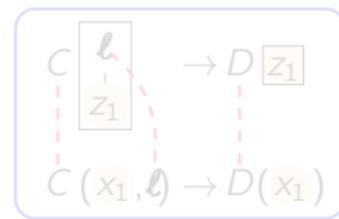
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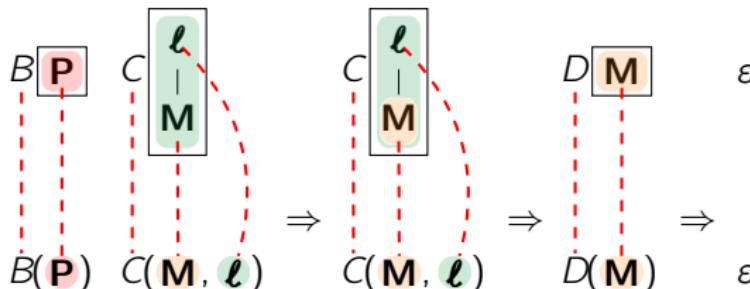
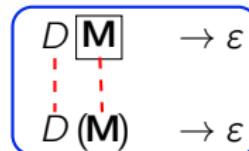
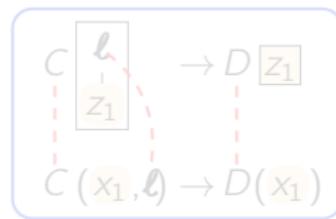
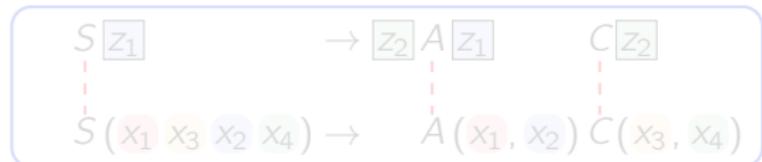
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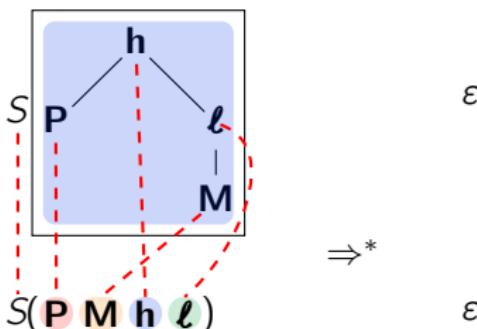
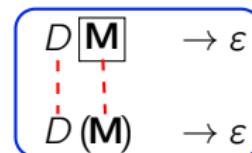
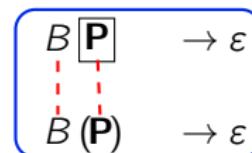
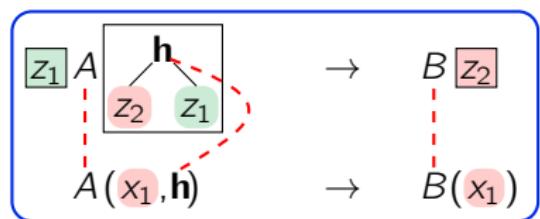
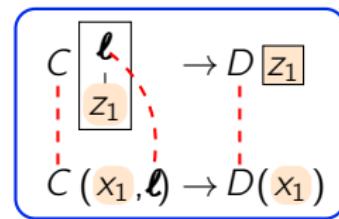
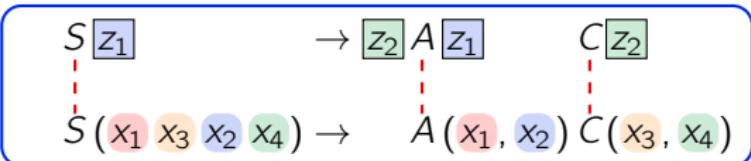
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E : natural language \mathcal{H} : set of hybrid trees

parsing : $E \rightarrow \mathcal{H}$

$$\text{parsing}(e) = \text{proj} \left(\underset{\substack{d \in D_G: \\ \text{str}(\text{proj}(d)) = e}}{\operatorname{argmax}} P(d \mid (G, p)) \right)$$

where

- ▶ (G, p) : probabilistic hybrid grammar
 $(D_G$: set of proof trees of G)
- ▶ $\text{proj} : D_G \rightarrow \mathcal{H}$
- ▶ $P(r_1 \dots r_n \mid (G, p)) = p(r_1) \cdot \dots \cdot p(r_n)$

parsing with hybrid grammar G :

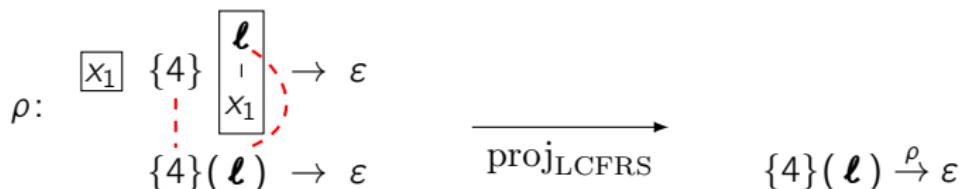
sentence $w = \mathbf{Jan}_1 \mathbf{Piet}_2 \mathbf{Marie}_3 \mathbf{zag}_4 \mathbf{helpen}_5 \mathbf{lezen}_6$

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2. parsing of w with $\text{proj}_{\text{LCFRS}}(G)$
⇒ proof tree d for “ $w \in L(\text{proj}_{\text{LCFRS}}(G))$ ”
3. enrich proof tree d by sDCP-part of used rules
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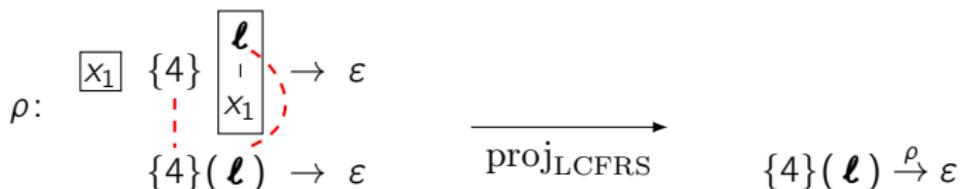
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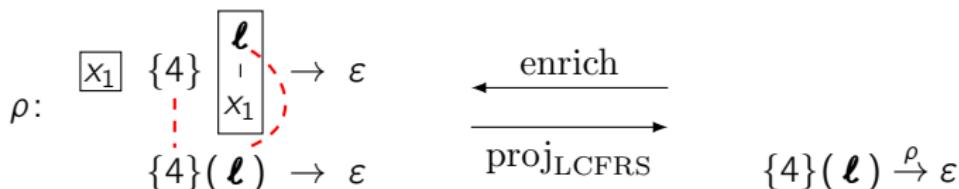
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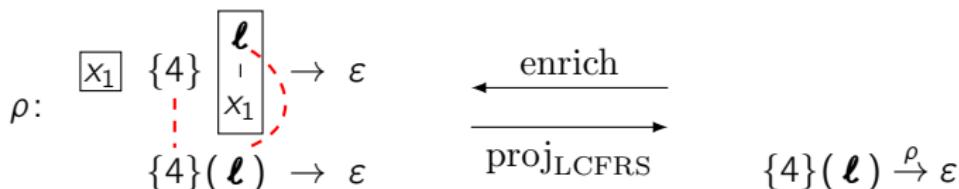
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How to obtain a hybrid grammar for E ?

- ▶ grammar induction from corpus of hybrid trees
[Nederhof, HV 14], [Gebhardt, Nederhof, HV 17]
- ▶ training of probabilities
[Drewes, Gebhardt, HV 16]
EM-training for weighted aligned hypergraph bimorphisms.
ACL Workshop StatFSM 2016

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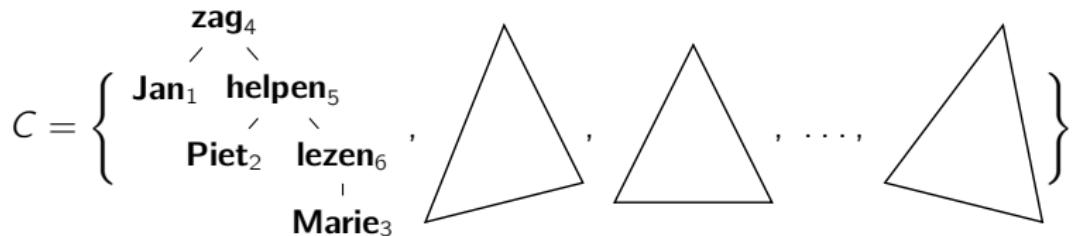
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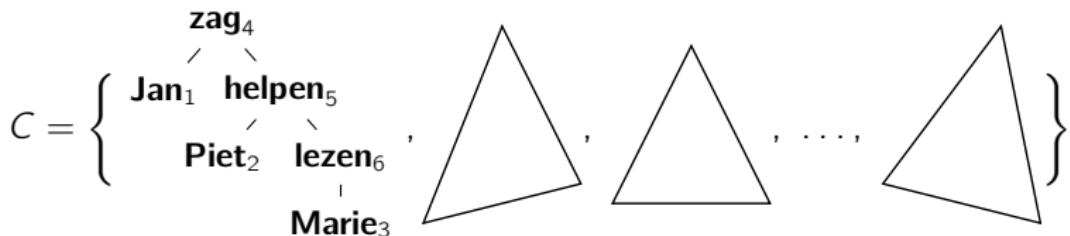
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corpus of dependency trees for language E :



(corpus = large, finite set)

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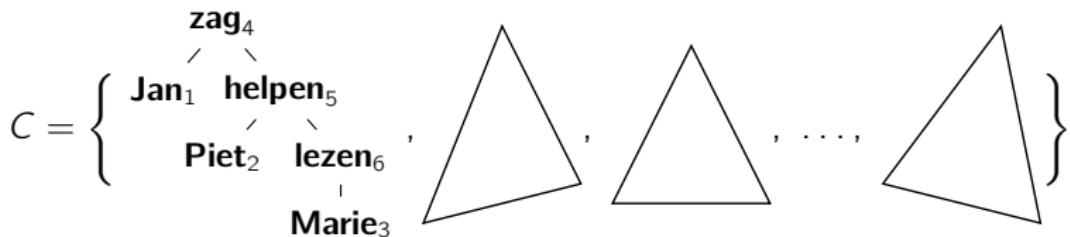


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corpora for dependency trees

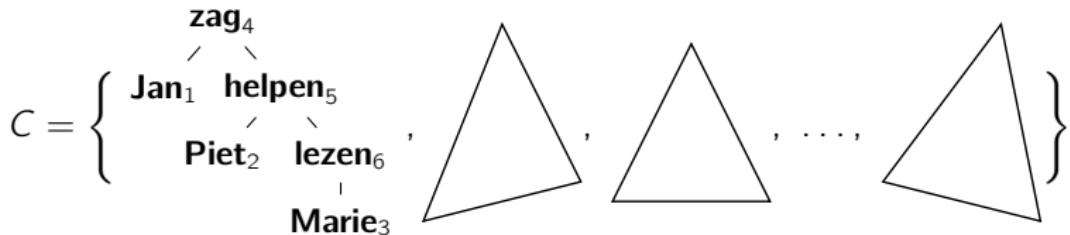
- ▶ Prague Dependency Treebank (Czech, 26k sentences)
- ▶ Slovene Dependency Treebank (Slovene, 30k words)
- ▶ Danish Dependency Treebank (Danish, 5k sentence)
- ▶ Talbanken05 (Swedish, 30k words)
- ▶ Metu-Sabancı treebank (Turkish, 7k sentences)
- ▶ ...

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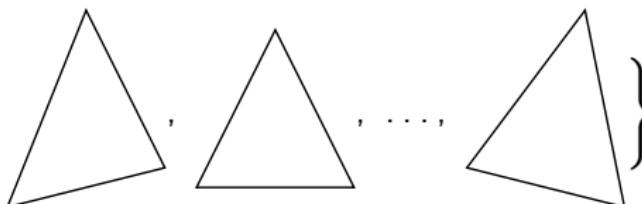
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construct hybrid grammar G such that $C \subsetneq L(G) \subsetneq \mathcal{H}$

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transformation $C \rightsquigarrow G$ is called **grammar induction**

grammar induction from one hybrid tree:

hybrid tree
 $h = (\xi, U, \preceq)$

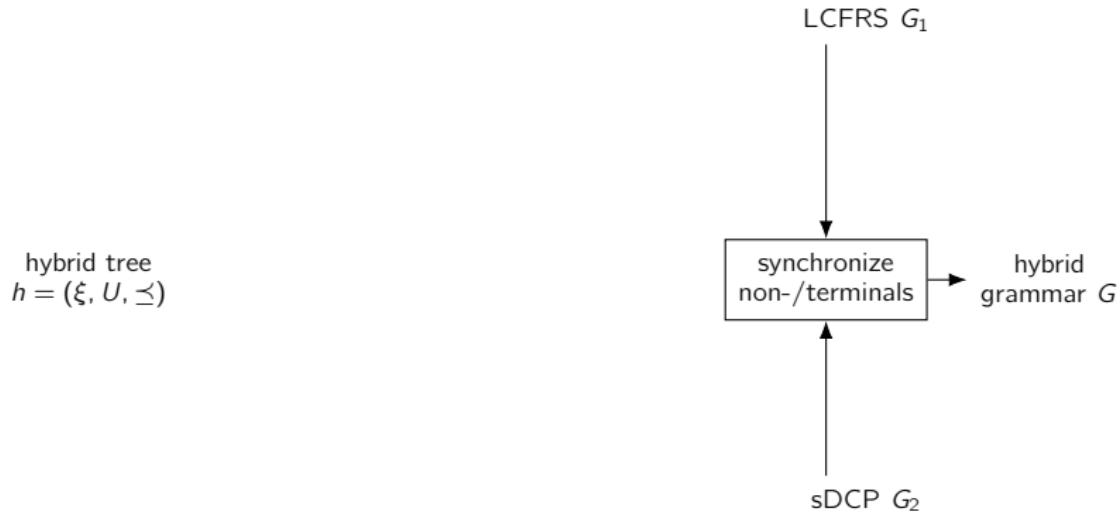
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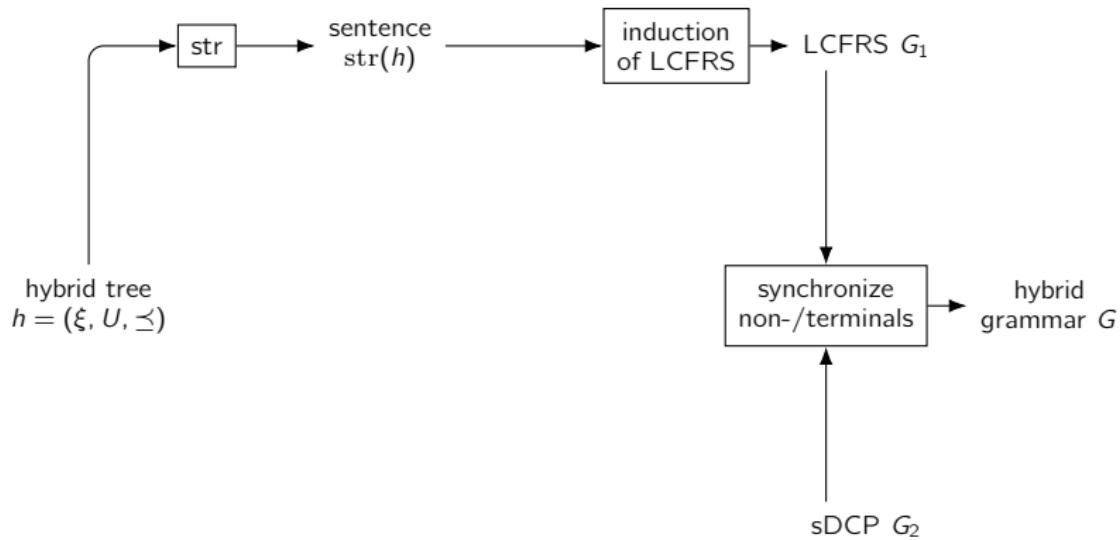
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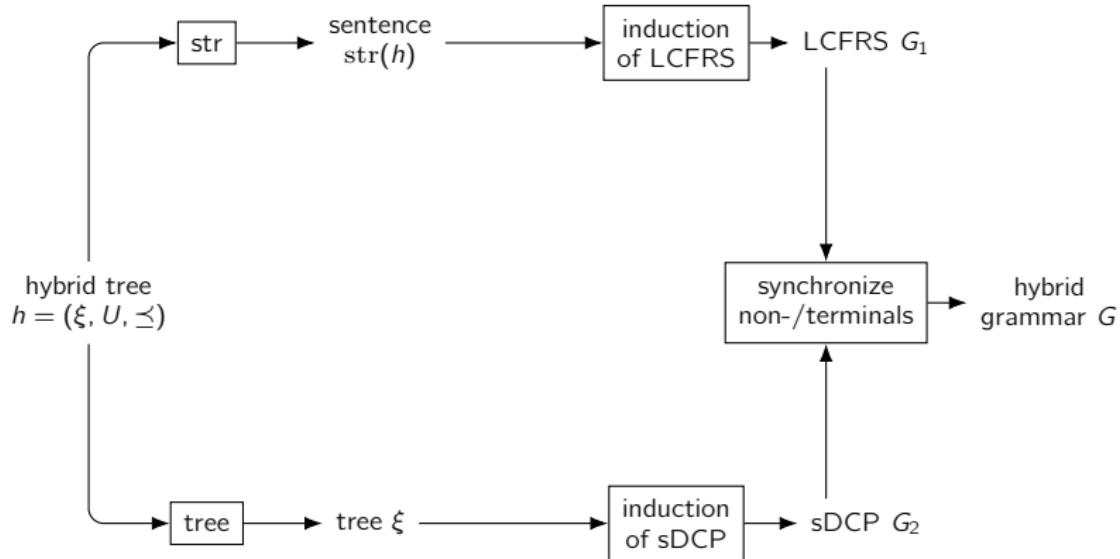
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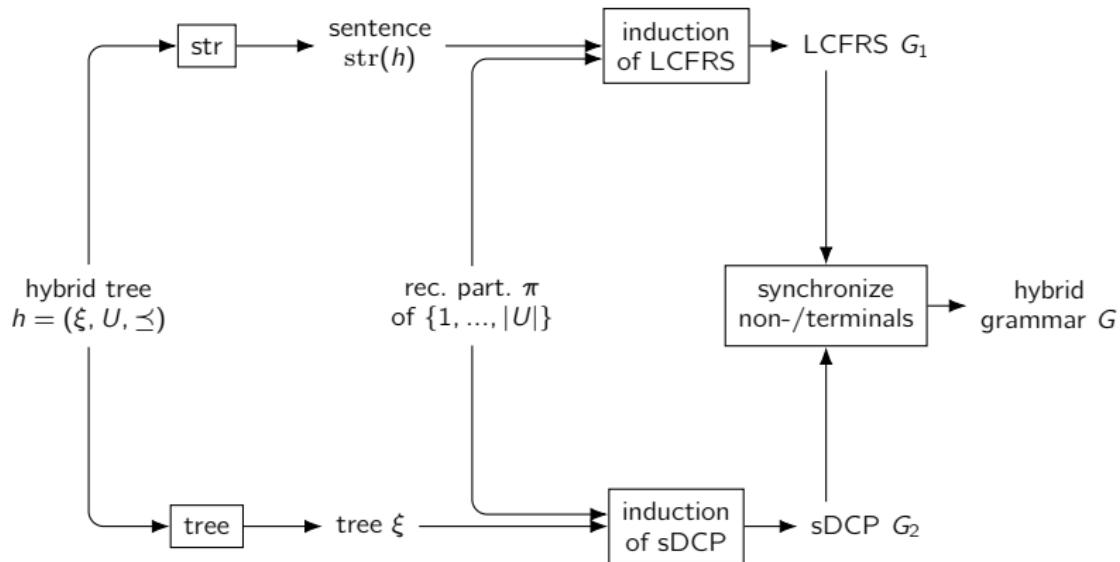
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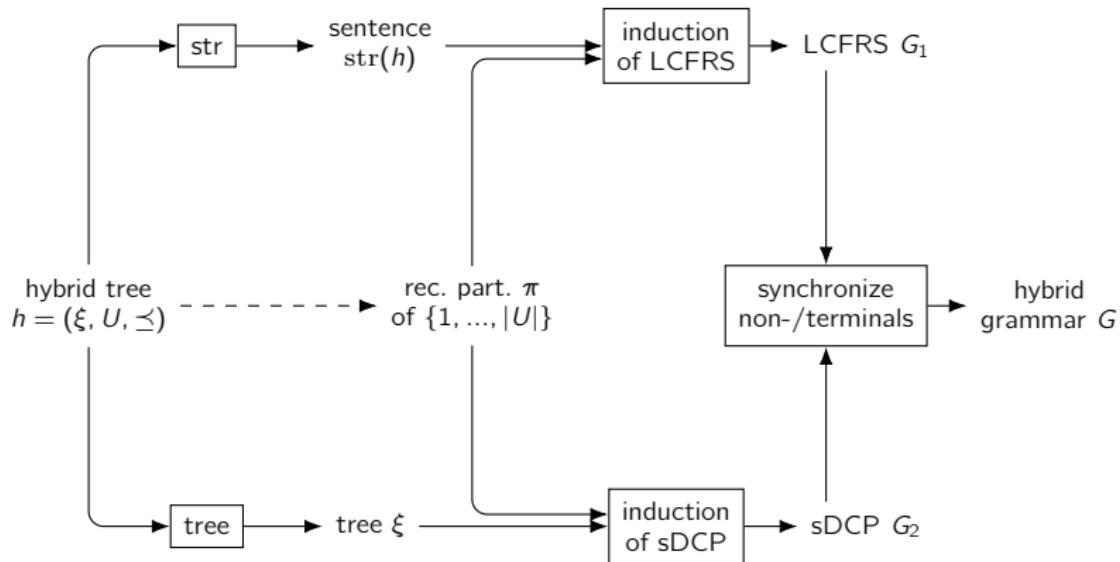
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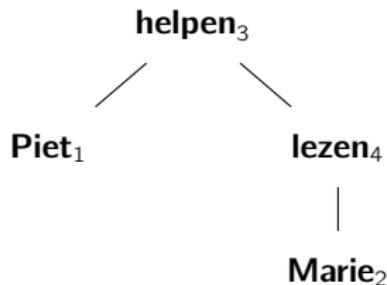
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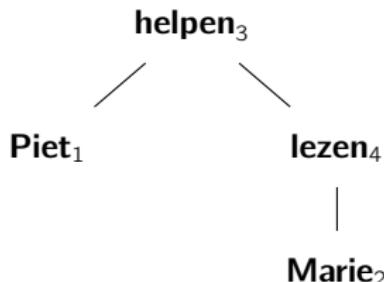
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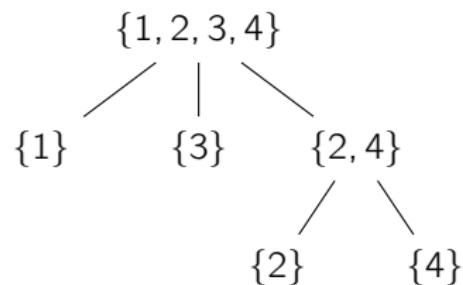
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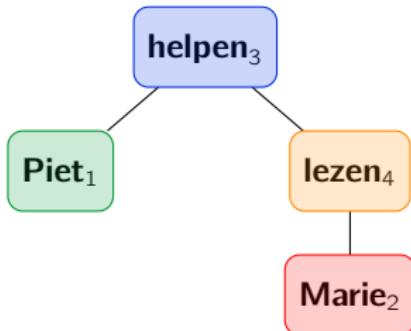


recursive partitioning π of $\{1, \dots, |U|\}$

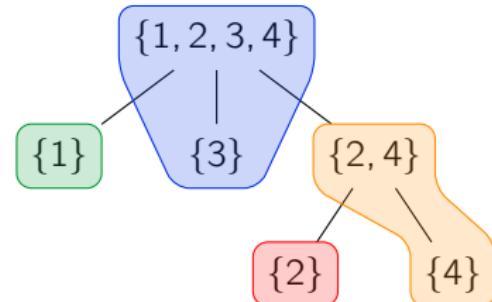


$$\text{fanout}(\pi) = \max_{p \in \text{pos}(\pi)} \left(\text{number of intervals in label of } \pi \text{ at } p \right)$$

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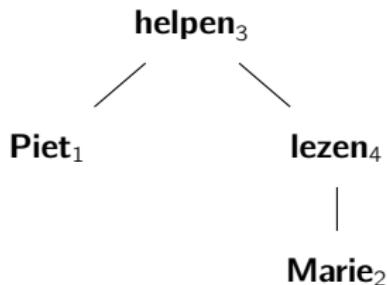
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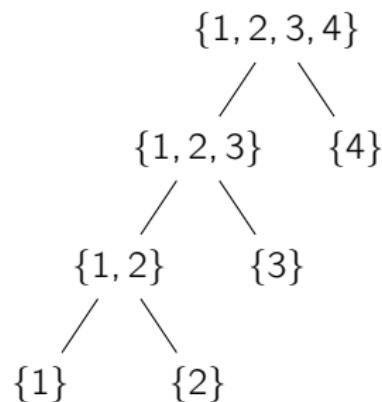
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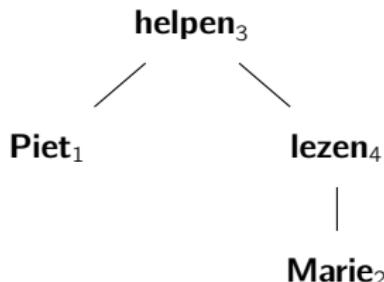
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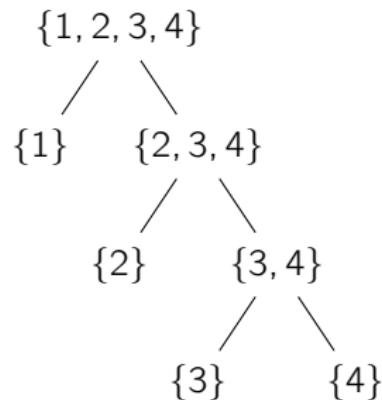
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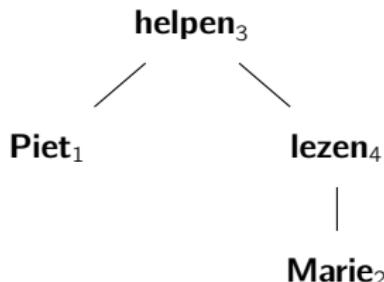
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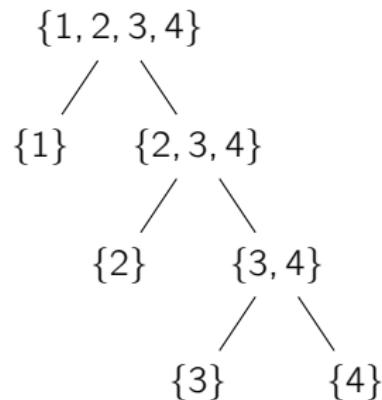
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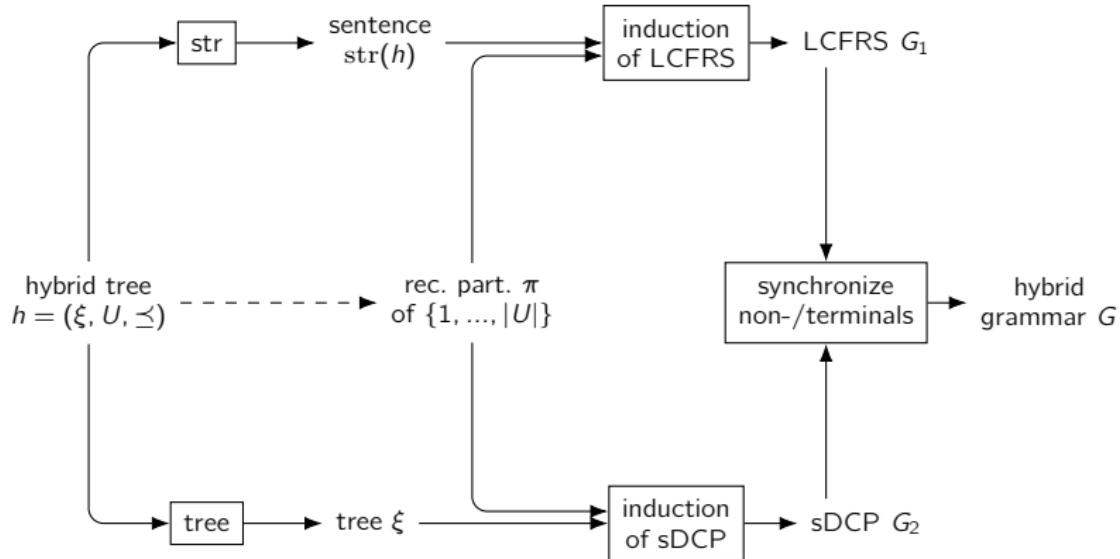
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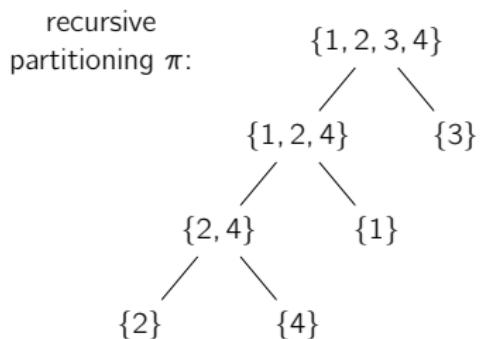
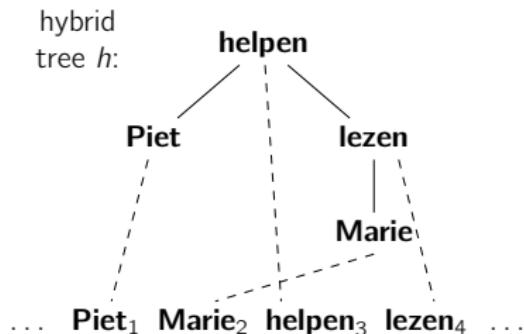
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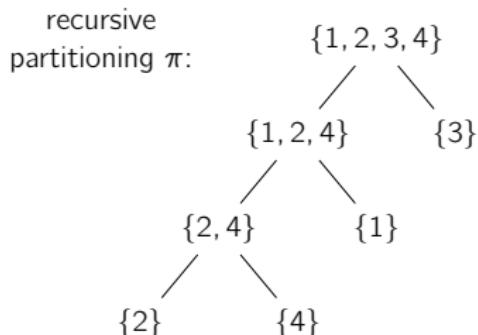
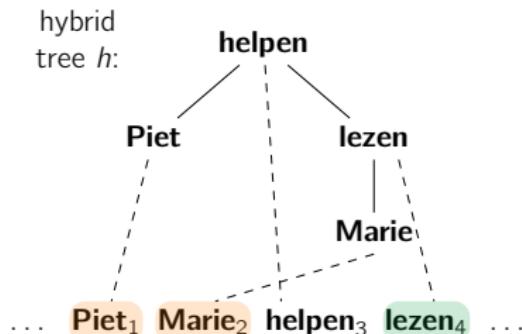
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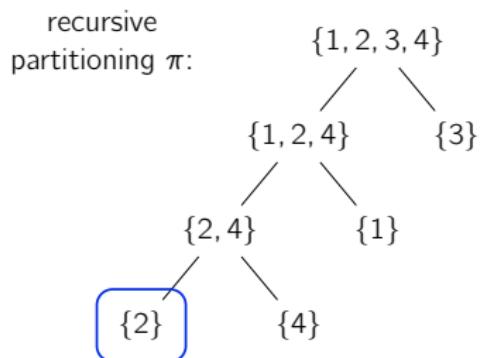
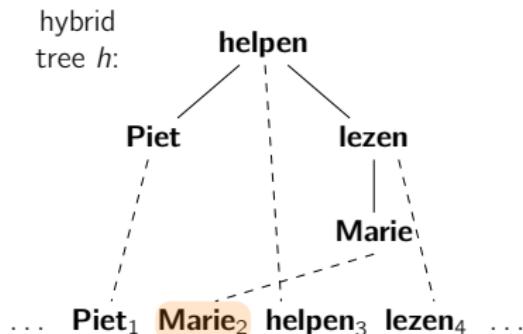


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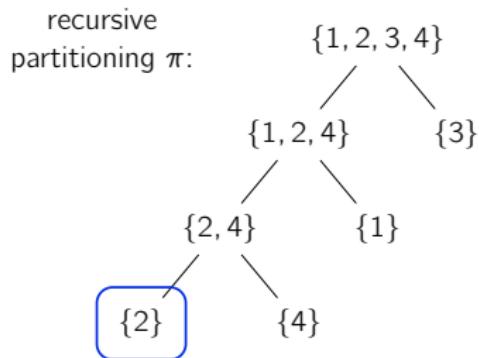
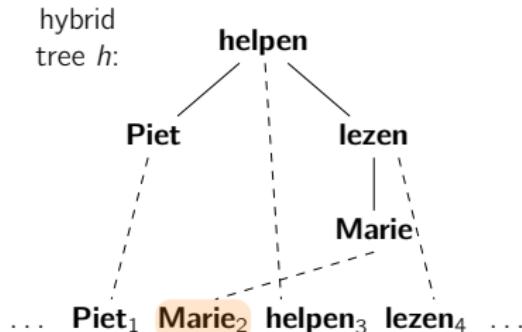


$$\{1, 2, 4\}(\text{Piet } \text{Marie}, \text{lezen}) \Rightarrow^* \varepsilon$$

induction of LCFRS G_1 :

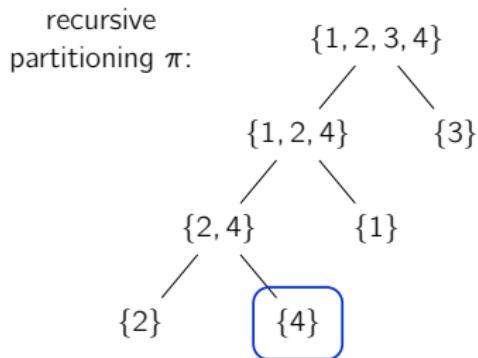
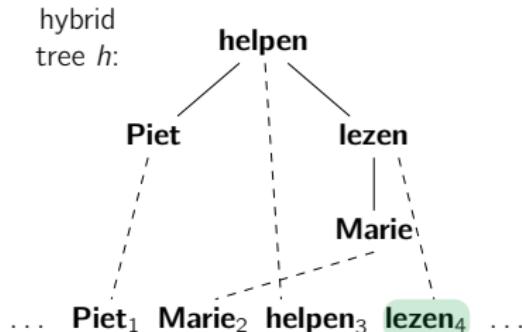


induction of LCFRS G_1 :



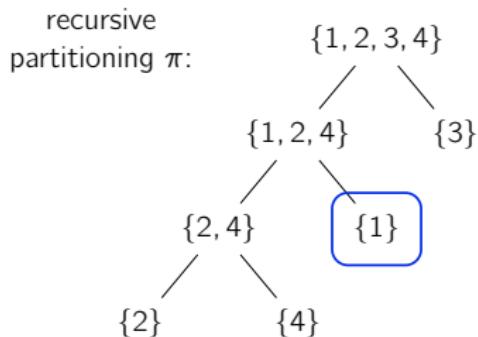
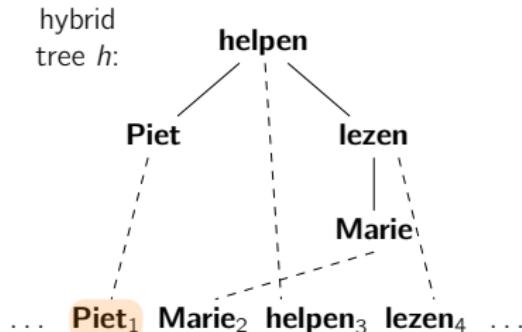
$$\{2\} (\mathbf{Marie}) \rightarrow \varepsilon$$

induction of LCFRS G_1 :



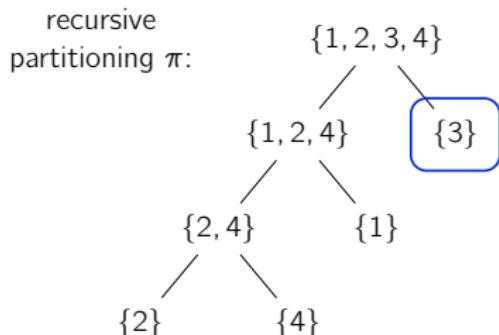
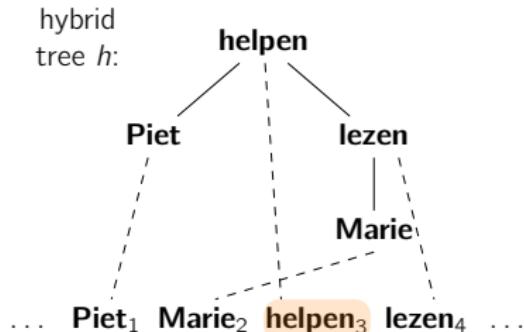
$$\{2\} (\mathbf{Marie}) \rightarrow \varepsilon \quad \{4\} (\mathbf{lezen}) \rightarrow \varepsilon$$

induction of LCFRS G_1 :



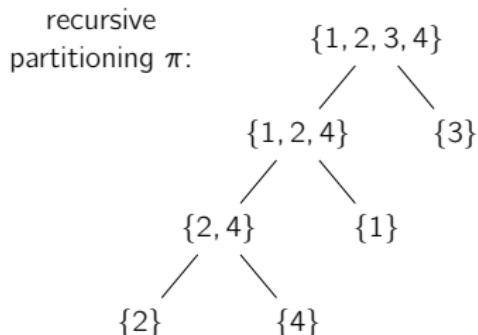
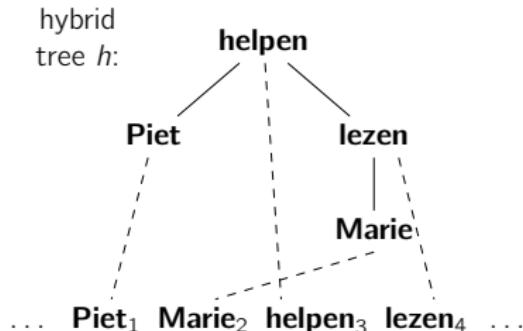
$$\{2\} (\mathbf{Marie}) \rightarrow \varepsilon \quad \{4\} (\mathbf{lezen}) \rightarrow \varepsilon \quad \{1\} (\mathbf{Piet}) \rightarrow \varepsilon$$

induction of LCFRS G_1 :



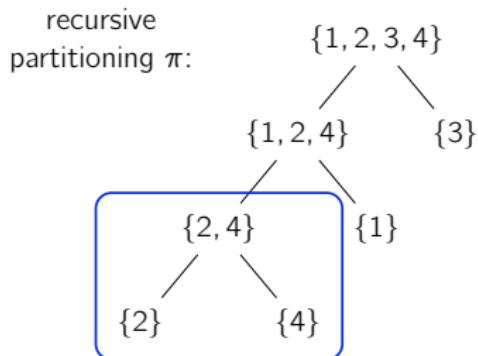
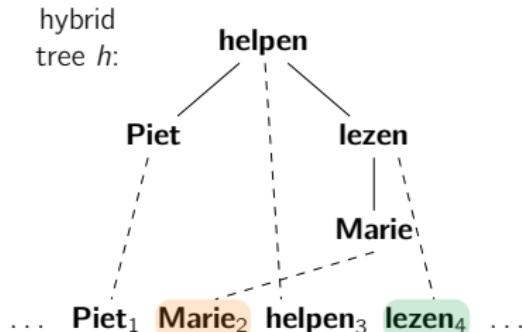
$$\{2\} (\mathbf{Marie}) \rightarrow \varepsilon \quad \{4\} (\mathbf{lezen}) \rightarrow \varepsilon \quad \{1\} (\mathbf{Piet}) \rightarrow \varepsilon \quad \{3\} (\mathbf{helpen}) \rightarrow \varepsilon$$

induction of LCFRS G_1 :



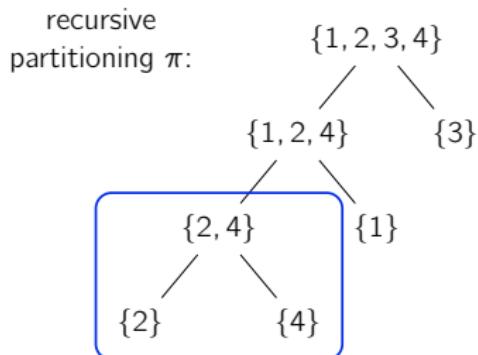
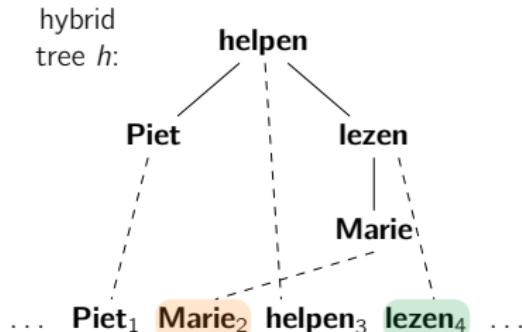
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$$\{2\} (\mathbf{Marie}) \rightarrow \varepsilon \quad \{4\} (\mathbf{lezen}) \rightarrow \varepsilon \quad \{1\} (\mathbf{Piet}) \rightarrow \varepsilon \quad \{3\} (\mathbf{helpen}) \rightarrow \varepsilon$$

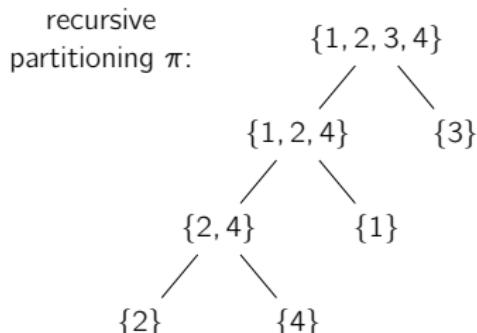
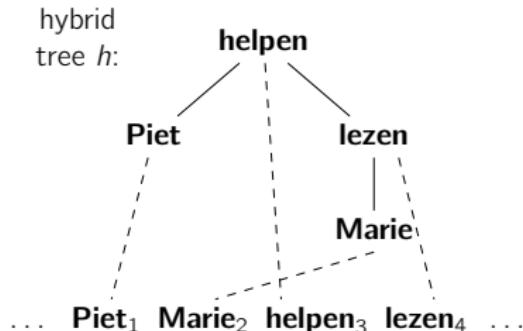
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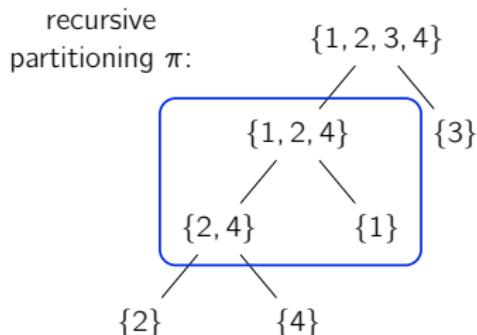
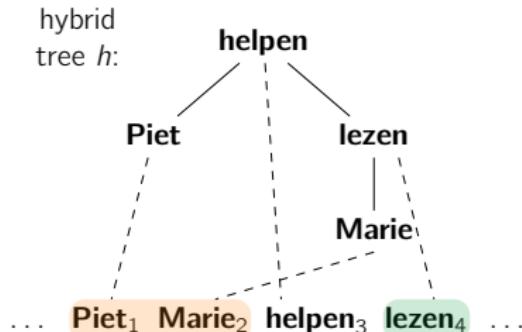
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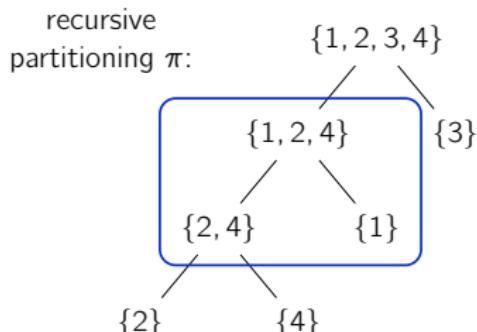
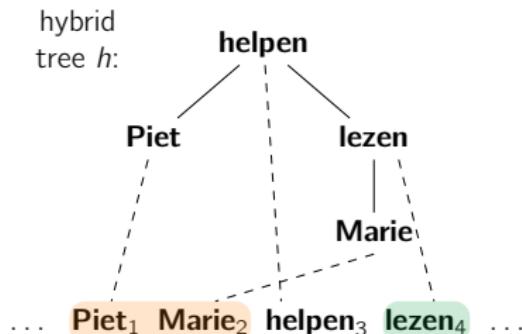
induction of LCFRS G_1 :



$$\{2\} (\mathbf{Marie}) \rightarrow \varepsilon \quad \{4\} (\mathbf{lezen}) \rightarrow \varepsilon \quad \{1\} (\mathbf{Piet}) \rightarrow \varepsilon \quad \{3\} (\mathbf{helpen}) \rightarrow \varepsilon$$

$$\{2, 4\}(x_1, x_2) \rightarrow \{2\}(x_1) \quad \{4\}(x_2)$$

induction of LCFRS G_1 :

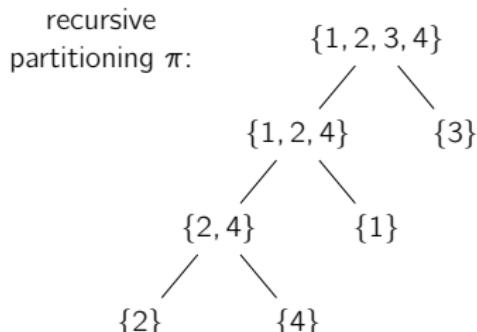
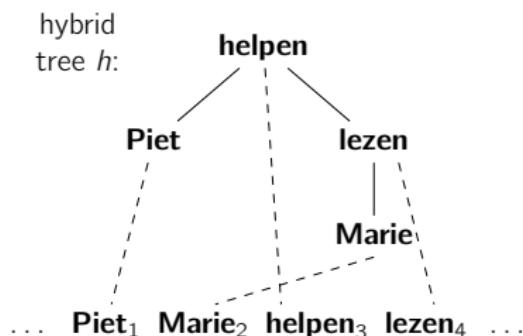


$$\{2\} (\mathbf{Marie}) \rightarrow \varepsilon \quad \{4\} (\mathbf{lezen}) \rightarrow \varepsilon \quad \{1\} (\mathbf{Piet}) \rightarrow \varepsilon \quad \{3\} (\mathbf{helpen}) \rightarrow \varepsilon$$

$$\{2, 4\}(x_1, x_2) \rightarrow \{2\}(x_1) \quad \{4\}(x_2)$$

$$\{1, 2, 4\}(x_3, x_1, x_2) \rightarrow \{2, 4\}(x_1, x_2) \quad \{1\}(x_3)$$

induction of LCFRS G_1 :

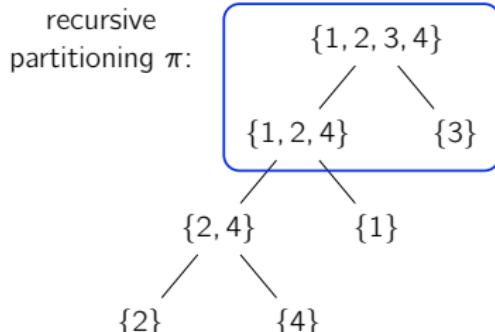
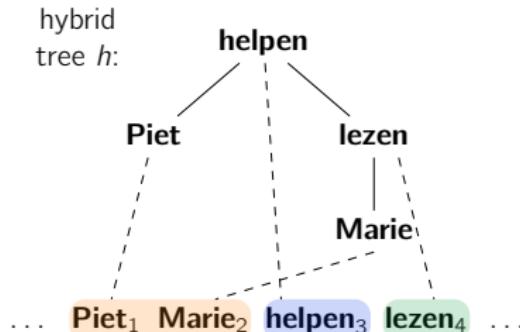


$$\{2\} (\mathbf{Marie}) \rightarrow \varepsilon \quad \{4\} (\mathbf{lezen}) \rightarrow \varepsilon \quad \{1\} (\mathbf{Piet}) \rightarrow \varepsilon \quad \{3\} (\mathbf{helpen}) \rightarrow \varepsilon$$

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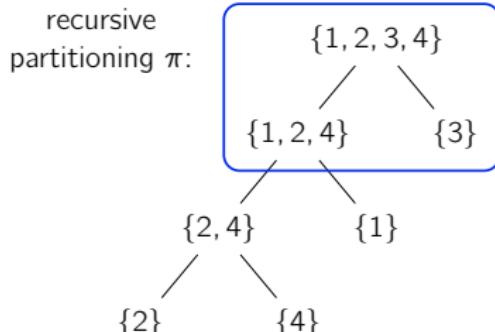
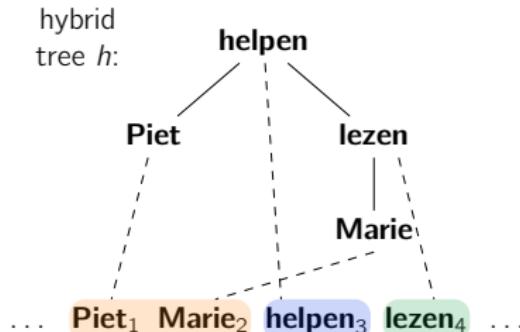


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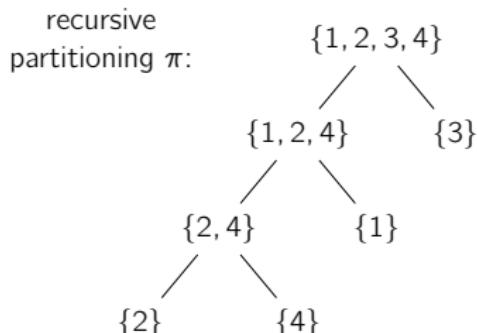
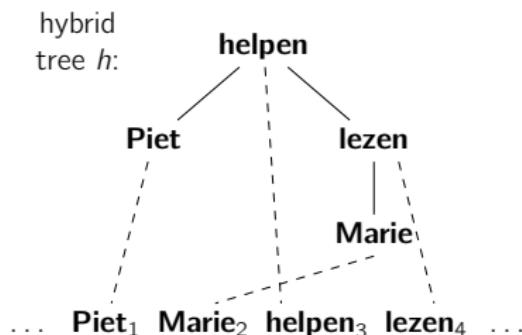
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$$\{2, 4\}(x_1, x_2) \rightarrow \{2\}(x_1) \quad \{4\}(x_2)$$

$$\{1, 2, 4\}(x_3, x_1, x_2) \rightarrow \{2, 4\}(x_1, x_2) \quad \{1\}(x_3)$$

$$\{1, 2, 3, 4\}(x_1, x_3, x_2) \rightarrow \{1, 2, 4\}(x_1, x_2) \quad \{3\}(x_3)$$

induction of LCFRS G_1 :



$$\{2\} (\mathbf{Marie}) \rightarrow \varepsilon \quad \{4\} (\mathbf{lezen}) \rightarrow \varepsilon \quad \{1\} (\mathbf{Piet}) \rightarrow \varepsilon \quad \{3\} (\mathbf{helpen}) \rightarrow \varepsilon$$

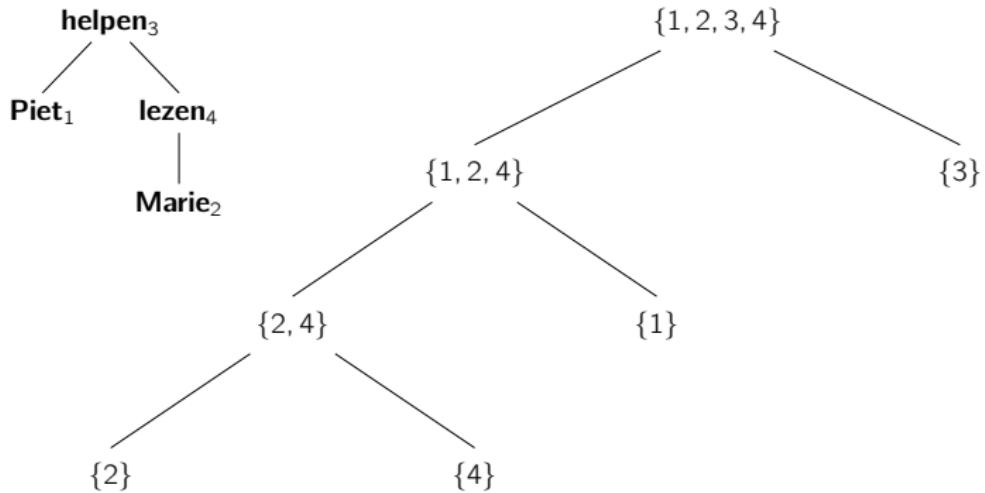
$$\{2, 4\}(x_1, x_2) \rightarrow \{2\}(x_1) \quad \{4\}(x_2)$$

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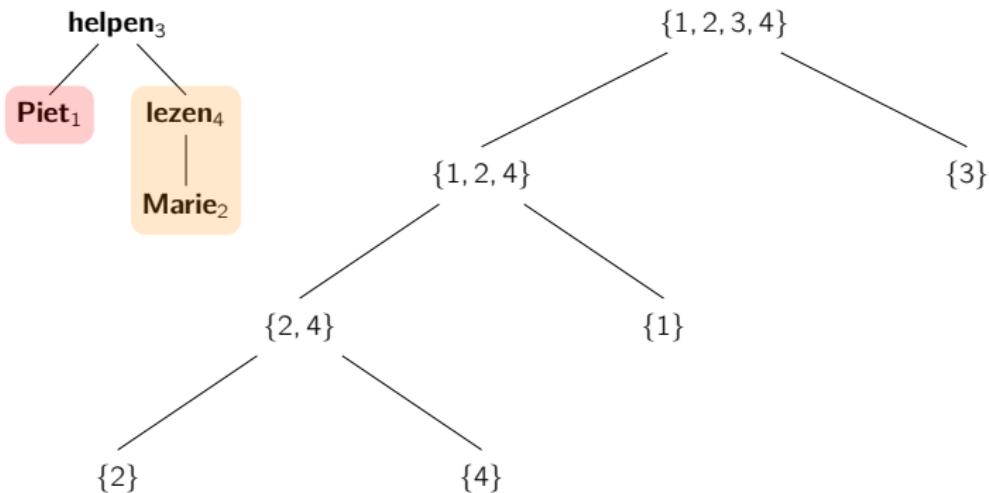
$$\{1, 2, 3, 4\}(x_1, x_3, x_2) \rightarrow \{1, 2, 4\}(x_1, x_2) \quad \{3\}(x_3)$$

$$\text{fanout}(G_1) = \text{fanout}(\pi)$$

induction of sDCP G_2 :

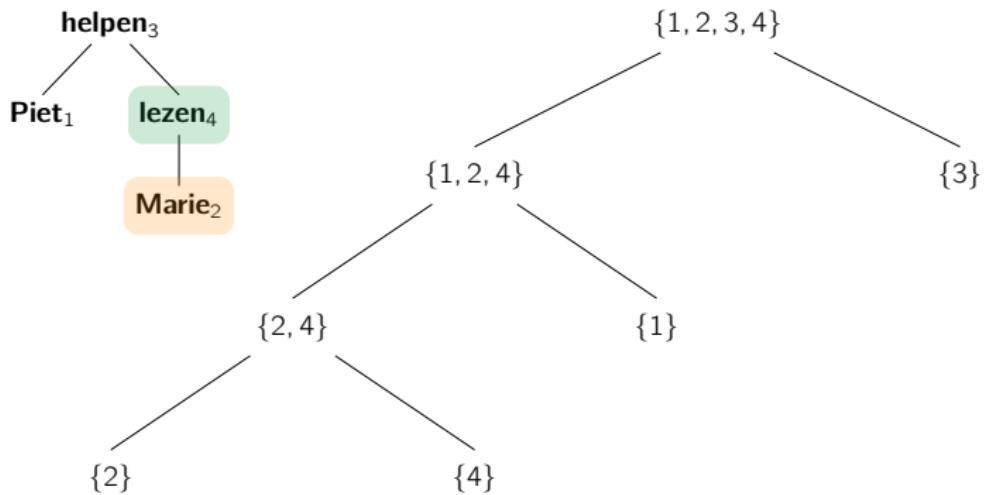


induction of sDCP G_2 :



$$\{1, 2, 4\} \textcolor{red}{P} \textcolor{brown}{\ell(\mathbf{M})} \Rightarrow^* \varepsilon$$

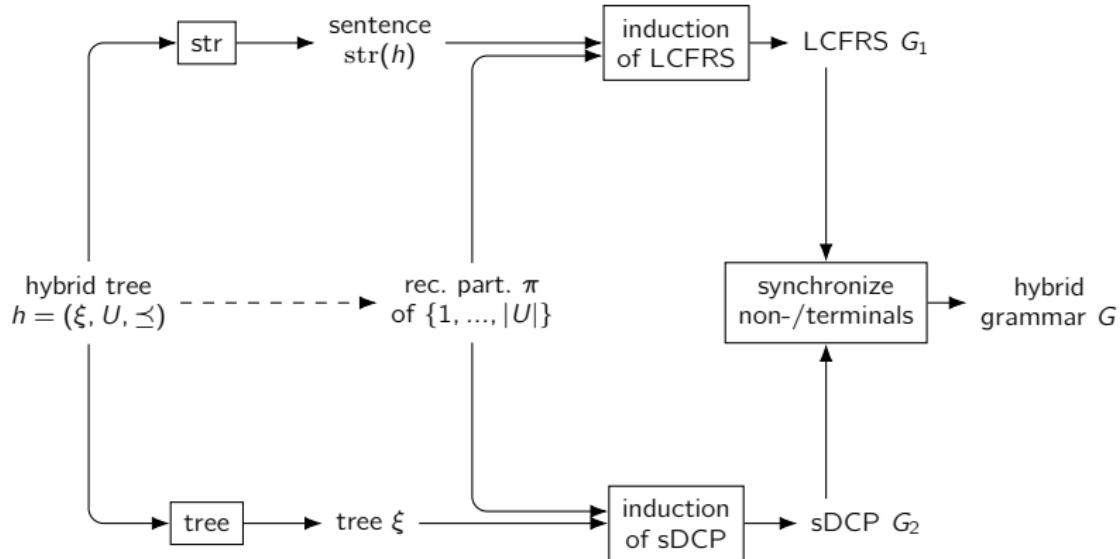
induction of sDCP G_2 :



$$\{1, 2, 4\} \boxed{\mathbf{P}} \boxed{\ell(\mathbf{M})} \Rightarrow^* \varepsilon$$

$$\boxed{\mathbf{M}} \{4\} \boxed{\ell(\mathbf{M})} \Rightarrow^* \varepsilon$$

grammar induction from one hybrid tree:



$$L(G) = \{h\}$$

parsing of $\text{str}(h)$ according to π

synchronization of nonterminals and terminals:

sDCP-rule: $\{2\} \quad \boxed{\mathbf{M}} \quad \rightarrow \quad \varepsilon$

LCFRS-rule: $\{2\} \quad (\mathbf{M}) \quad \rightarrow \quad \varepsilon$

synchronization of nonterminals and terminals:

sDCP-rule:

$$\{2\} \quad \boxed{\mathbf{M}} \quad \rightarrow \quad \varepsilon$$

LCFRS-rule:

$$\{2\} \quad (\mathbf{M}) \quad \rightarrow \quad \varepsilon$$

synchronization of nonterminals and terminals:

sDCP-rule:

$$\{2\} \quad \boxed{\mathbf{M}} \quad \rightarrow \quad \varepsilon$$

LCFRS-rule:

$$\{2\} \quad (\mathbf{M}) \quad \rightarrow \quad \varepsilon$$

synchronization of nonterminals and terminals:

sDCP-rule:

$$\{2\} \boxed{\mathbf{M}} \rightarrow \varepsilon$$

$$\boxed{x_1} \{4\} \boxed{\begin{matrix} \ell \\ | \\ x_1 \end{matrix}} \rightarrow \varepsilon$$

LCFRS-rule:

$$\{2\} (\mathbf{M}) \rightarrow \varepsilon$$

$$\{4\}(\ell) \rightarrow \varepsilon$$

synchronization of nonterminals and terminals:

sDCP-rule:

$$\{2\} \boxed{\mathbf{M}} \rightarrow \varepsilon$$

$$\boxed{x_1} \quad \{4\} \boxed{\ell} \quad | \quad x_1 \rightarrow \varepsilon$$

LCFRS-rule:

$$\{2\} \quad (\mathbf{M}) \rightarrow \varepsilon$$

$$\{4\}(\ell) \rightarrow \varepsilon$$

synchronization of nonterminals and terminals:

sDCP-rule:

$$\{2\} \boxed{\mathbf{M}} \rightarrow \varepsilon$$

$$x_1 \{4\} \boxed{\ell} \rightarrow \varepsilon$$

LCFRS-rule:

$$\{2\} (\mathbf{M}) \rightarrow \varepsilon$$

$$\{4\}(\ell) \rightarrow \varepsilon$$

synchronization of nonterminals and terminals:

sDCP-rule:

$$\{2\} \boxed{\mathbf{M}} \rightarrow \varepsilon$$

$$\boxed{x_1} \quad \{4\} \boxed{\ell} \quad | \quad \boxed{x_1} \rightarrow \varepsilon$$

LCFRS-rule:

$$\{2\} (\mathbf{M}) \rightarrow \varepsilon$$

$$\{4\} (\ell) \rightarrow \varepsilon$$

sDCP-rule:

$$\{2, 4\} \boxed{x_2} \rightarrow \{2\} \boxed{x_1} \quad \boxed{x_1} \{4\} \boxed{x_2}$$

LCFRS-rule:

$$\{2, 4\} (x_1, x_2) \rightarrow \{2\} (x_1) \quad \{4\} (x_2)$$

synchronization of nonterminals and terminals:

sDCP-rule:

$$\{2\} \boxed{\mathbf{M}} \rightarrow \varepsilon$$

$$\boxed{x_1} \{4\} \boxed{\ell} \mid \boxed{x_1} \rightarrow \varepsilon$$

LCFRS-rule:

$$\{2\} (\mathbf{M}) \rightarrow \varepsilon$$

$$\{4\} (\ell) \rightarrow \varepsilon$$

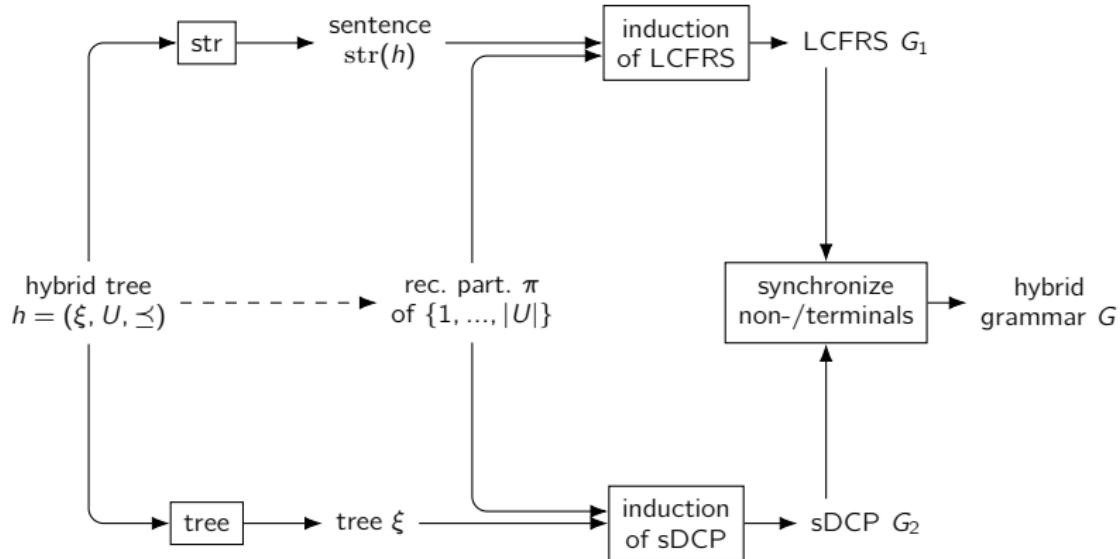
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grammar induction from one hybrid tree:

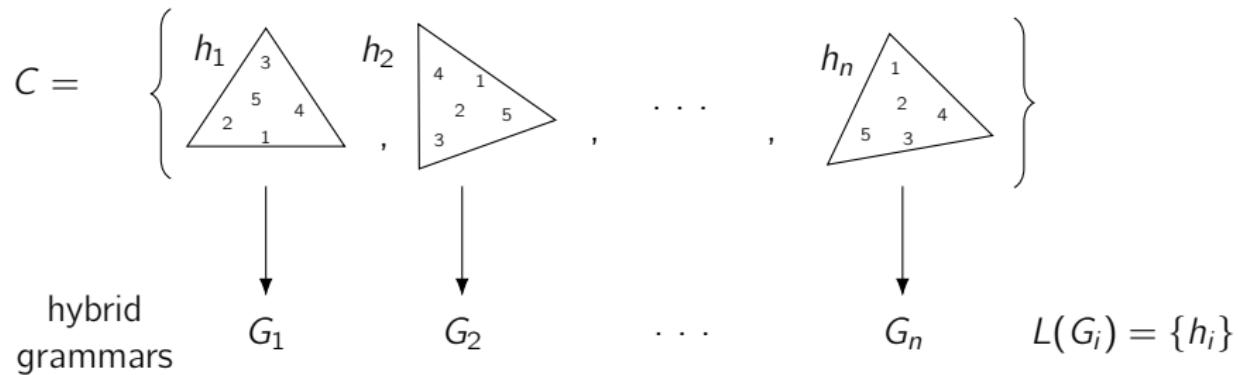


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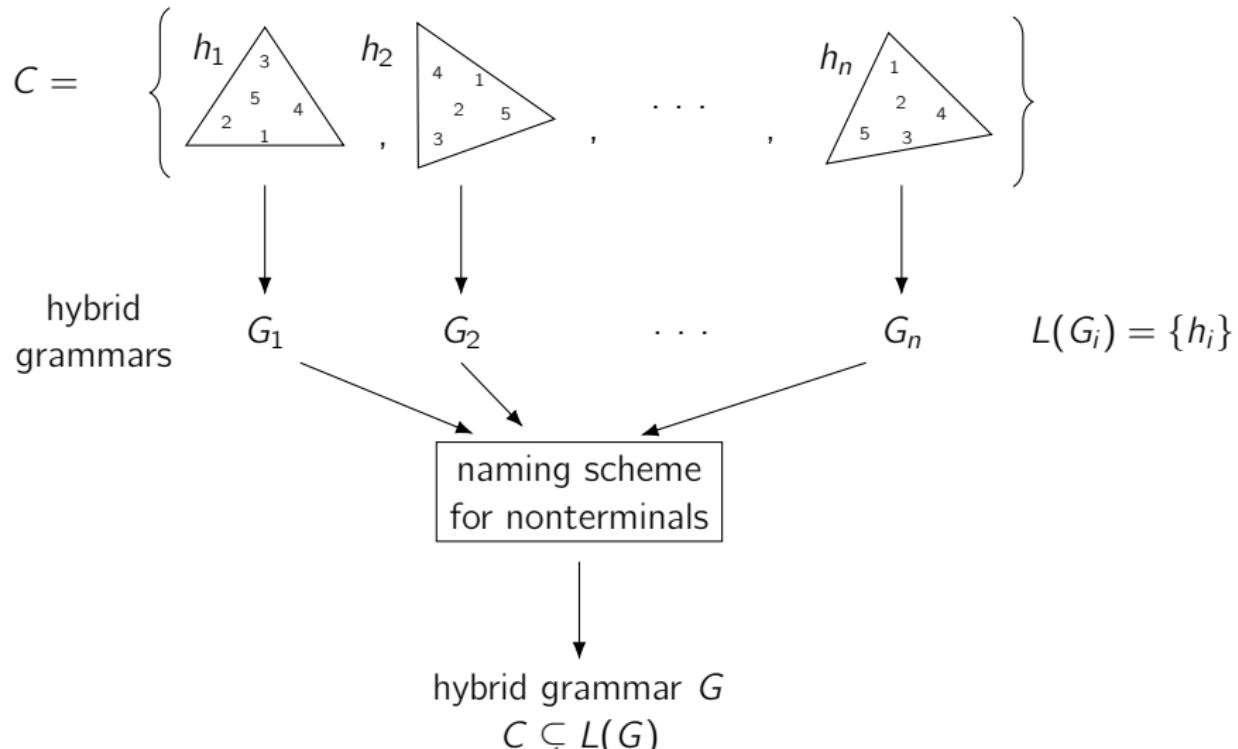
grammar induction from a corpus of hybrid trees:

corpus of hybrid trees + choice of recursive partitioning



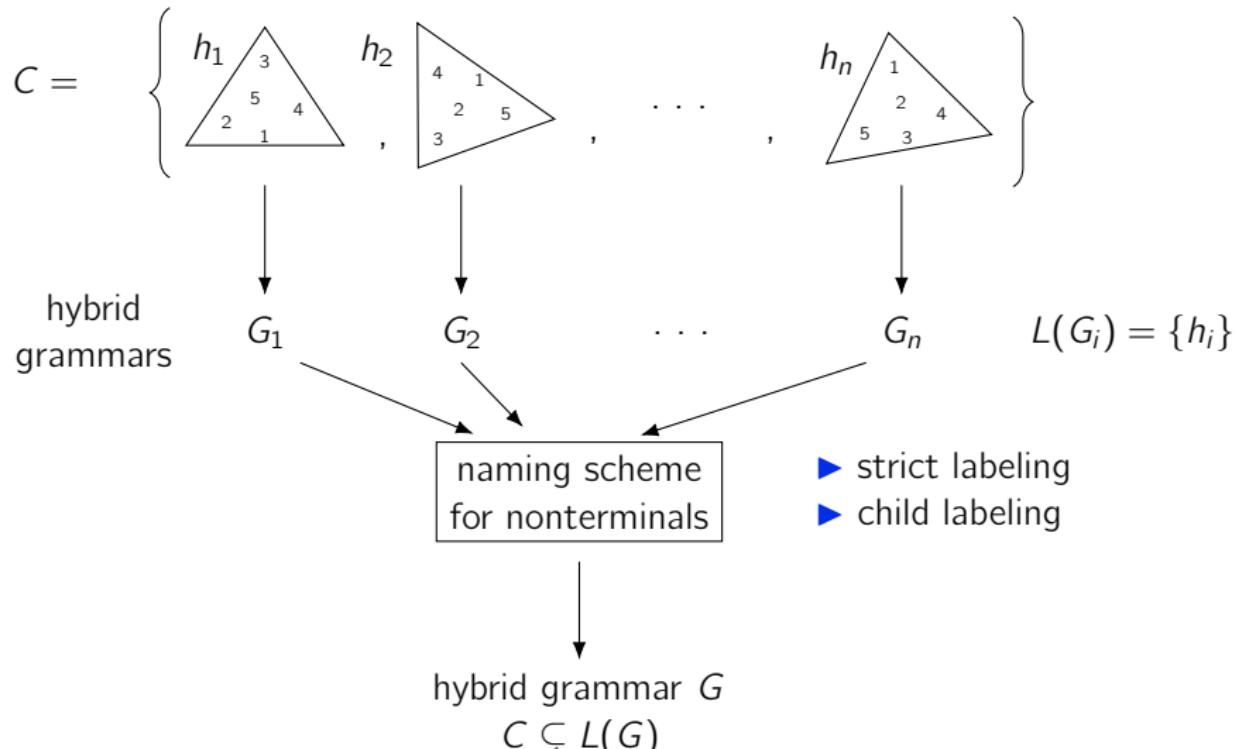
grammar induction from a corpus of hybrid trees:

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grammar formalisms	phrase structure trees		dependency trees		parsing compl.
	cont.	discont.	proj.	non-proj.	
	-	-	-	-	
REG/HMM	-	-	-	-	$\mathcal{O}(n)$
CFG	x	-	x	-	$\mathcal{O}(n^3)$ (CNF)
LCFRS	x	x	x	x	$\mathcal{O}(n^{3 \cdot \text{fanout}(G)})$ (binar.)

(LCFRS,sDCP)-hybrid grammar:



choice of recursive partitioning:

- directly extracted and transformed to fanout $\leq k$ $\quad \mathcal{O}(n^{3 \cdot k})$
- left-/right-branching $\quad \mathcal{O}(n)$

experiments for dependency parsing:

- ▶ NEGRA corpus; 16.509 German language sentences of length ≤ 25

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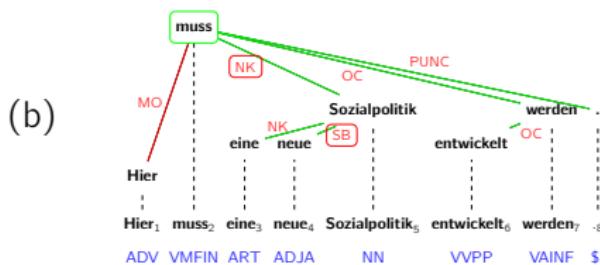
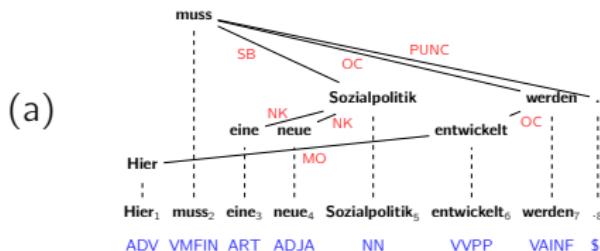
- ▶ NEGRA corpus; 16.509 German language sentences of length ≤ 25
- ▶ split into: training corpus (14.858) and test corpus (1.651)

training corpus $C_1 \longrightarrow$ probabilistic hybrid grammar (G, p)

test corpus C_2 : $\forall h \in C_2 : h \stackrel{?}{=} \text{parsing}_{(G,p)}(\text{str}(h))$

experiments for dependency parsing:

- ▶ NEGRA corpus; 16.509 German language sentences of length ≤ 25
- ▶ split into: training corpus (14.858) and test corpus (1.651)
- ▶ evaluation metrics:
unlabeled attachment score (UAS), labeled attachment score (LAS),
label accuracy (LA)



UAS: 7 / 8

LAS: 5 / 8

LA: 6 / 8

experiments for dependency parsing:

- ▶ NEGRA corpus; 16.509 German language sentences of length ≤ 25
- ▶ split into: training corpus (14.858) and test corpus (1.651)
- ▶ evaluation metrics:
unlabeled attachment score (UAS), labeled attachment score (LAS),
label accuracy (LA)
- ▶ parameters of our grammar induction:
 - ▶ naming scheme (strict labeling, child labeling)
 - ▶ argument label (POS+DEPREL, POS, DEPREL)
 - ▶ recursive partitioning (directly extracted, transformed to fanout k ,
left-/right-branching)

rec.	part.	argument label	nont.	rules	f_{\max}	f_{avg}	fail	UAS	LAS	LA	time
child labeling											
direct	POS+DEPREL	4,739	27,042	7		1.10	693	52.2	40.8	42.6	253
$k = 1$	POS+DEPREL	13,178	35,071	1		1.00	202	77.0	69.1	73.3	288
$k = 2$	POS+DEPREL	11,156	32,231	2		1.17	195	77.7	70.1	74.2	355
r-branch	POS+DEPREL	42,577	79,648	1		1.00	775	45.7	32.8	34.7	49
l-branch	POS+DEPREL	40,100	75,321	1		1.00	768	46.0	33.2	35.0	45
direct	POS	675	19,276	7		1.24	303	69.3	50.0	55.4	300
$k = 1$	POS	3,464	15,826	1		1.00	30	82.5	63.5	70.6	244
$k = 2$	POS	2,099	13,347	2		1.40	35	82.4	63.3	70.5	410
r-branch	POS	19,804	51,733	1		1.00	372	62.7	44.7	50.6	222
l-branch	POS	17,240	45,883	1		1.00	342	63.9	45.6	51.4	197
direct	DEPREL	2,505	19,511	7		1.13	3	78.9	71.6	78.6	484
$k = 1$	DEPREL	8,059	22,613	1		1.00	1	79.5	71.7	79.0	608
$k = 2$	DEPREL	6,651	20,314	2		1.20	1	79.8	72.0	79.2	971
$k = 3$	DEPREL	6,438	19,962	3		1.25	1	79.5	71.9	79.1	1,013
r-branch	DEPREL	27,653	54,360	1		1.00	2	76.3	67.5	76.1	216
l-branch	DEPREL	25,699	50,418	1		1.00	1	76.2	67.6	76.1	198
cascade: child labeling, $k = 1$, POS+DEPREL/POS/DEPREL						1	84.3	76.1	81.6		325
LCFRS [Maier, Kallmeyer 10]:											
rparse simple		920	18,587	7		1.37	56	77.3	70.0	76.2	350
rparse ($v = 1, h = 3$)		40,141	61,450	7		1.10	13	78.5	71.4	79.0	778
MaltParser, unlexicalized, stacklazy [Nivre, Hall, Nilsson 06]						0	85.6	80.0	85.0		24

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LCFRS [Maier, Kallmeyer 10]:

rparse simple	920	18,587	7	1.37	56	77.3	70.0	76.2	350
rparse ($v = 1, h = 3$)	40,141	61,450	7	1.10	13	78.5	71.4	79.0	778

MaltParser, unlexicalized, stacklazy [Nivre, Hall, Nilsson 06]

0 85.6 80.0 85.0 24

[Chomsky 56]: Three Models for the Description of Language.

*"There are **two central problems** in the descriptive study of languages. One primary concern of the linguist is **to discover simple and *revealing* grammars for natural languages**. At the same time, **by studying the properties of such successful grammars ...**, he hopes to arrive at a general theory of linguistic structure."*

parsing of natural languages with ...

- ▶ probabilistic LCFRS
- ▶ probabilistic hybrid grammars
 - ▶ induction of probabilistic (LCFRS,sDCP)-hybrid grammars
 - ▶ experiments

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thanks!

References:

- ▶ [Chomsky 56] Three Models for the Description of Language.
- ▶ [Deransart, Maluszynski 85] Relating logic programs and attribute grammars.
- ▶ [Drewes, Gebhardt, HV 16] EM-training for weighted aligned hypergraph bimorphisms.
- ▶ [Gebhardt, Nederhof, HV 17] Hybrid grammars for parsing of discontinuous phrase structures and non-projective dependency structures.
- ▶ [Kallmeyer, Maier 10] Data-driven parsing with probabilistic linear context-free rewriting systems.
- ▶ [Kuhlmann, Satta 09] Treebank grammar techniques for non-projective dependency parsing.
- ▶ [Maier, Kallmeyer 10] Discontinuity and non-projectivity: Using mildly context-sensitive formalisms for data-driven parsing.
- ▶ [Nivre, Hall, Nilsson 06] Maltparser: A data-driven parser-generator for dependency parsing.
- ▶ [Rounds 70] Tree-oriented proofs of some theorems on context-free and indexed languages.
- ▶ [Salomaa, Soittola 78] Automata-Theoretic Aspects of Formal Power Series.
- ▶ [Seki, Matsumura, Fujii, Kasami 91] On multiple context-free grammars.
- ▶ [Tesnière 59] Éléments de syntaxe structurale.
- ▶ [Vijay-Shanker, Weir, Joshi 87] Characterizing structural descriptions produced by various grammatical formalisms.

... a bit of theory of LCFRS:

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- ▶ Chomsky-Schützenberger theorem for weighted LCFRS

synchronous grammars:

(two grammars of the same type; synchronization via nonterminals)

- ▶ transduction grammars [Lewis, Stearns 68]
= synchronous context-free grammars [Chiang 07]
- ▶ generalized syntax-directed translation schemes [Aho, Ullman 71]
- ▶ synchronous tree-substitution grammars [Schabes 90]
- ▶ synchronous tree-adjoining grammars [Abeillé, Schabes, Joshi 90; Shieber, Schabes 90]
- ▶ synchronous context-free tree grammars [Nederhof, V. 12]
- ▶ synchronous linear context-free rewriting systems [Kaeshammer 13]

corpora for phrase structure trees

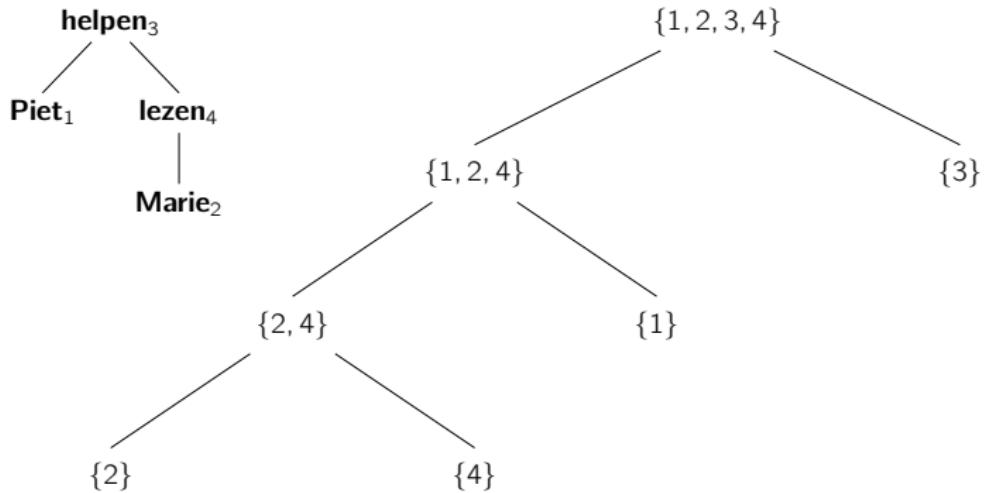
corpus = large, finite set

- ▶ TIGER treebank (German, 50k sentences)
- ▶ NEGRA corpus (German, 20k sentences)
- ▶ Japanese Verbmobil treebank (Japanese, 20k sentences)
- ▶ The Bosque part of the Floresta sintà(c)tica (Portuguese, 41k sentences)
- ▶ Alpino treebank (Dutch, 150k words)
- ▶ Sinica treebank (Chinese, 361k words)
- ▶ ...

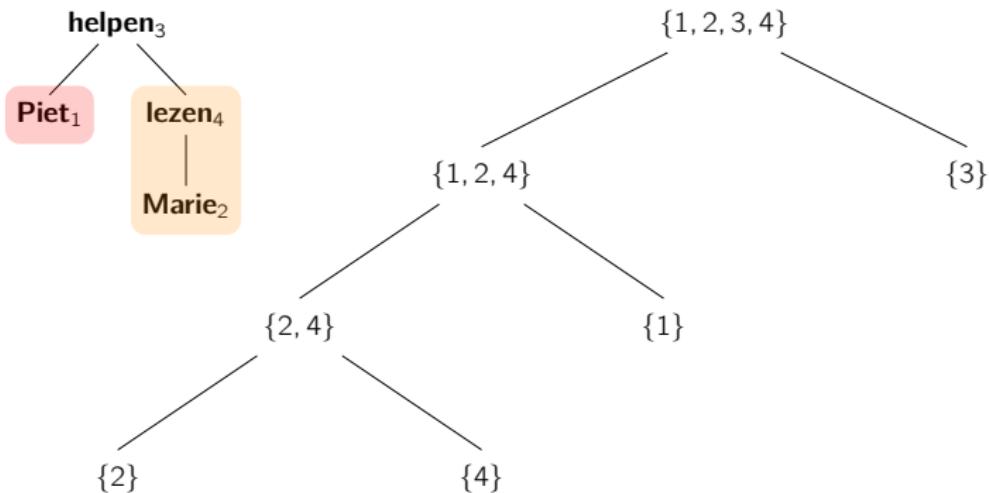
corpora for dependency trees

- ▶ Prague Dependency Treebank (Czech, 26k sentences)
- ▶ Slovene Dependency Treebank (Slovene, 30k words)
- ▶ Danish Dependency Treebank (Danish, 5k sentence)
- ▶ Talbanken05 (Swedish, 30k words)
- ▶ Metu-Sabancı treebank (Turkish, 7k sentences)
- ▶ ...

induction of sDCP G_2 :

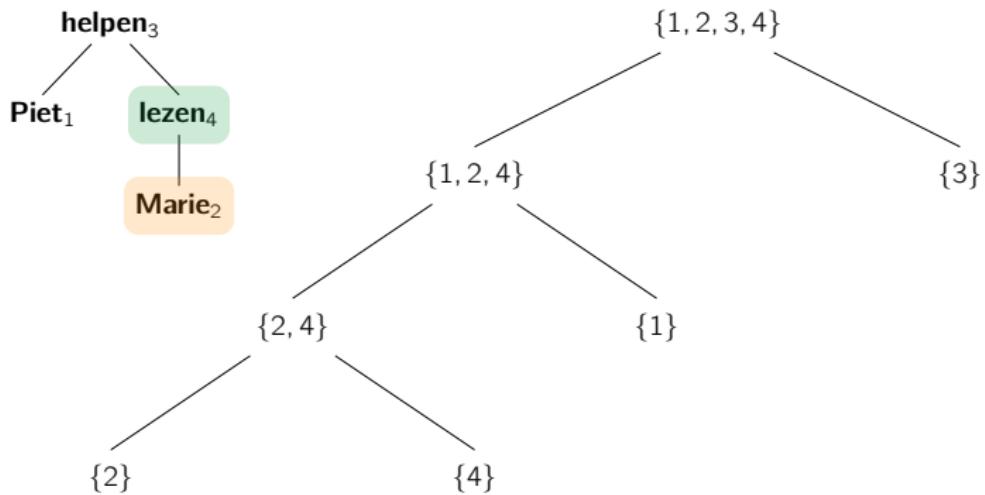


induction of sDCP G_2 :



$$\{1, 2, 4\} \textcolor{red}{P} \textcolor{brown}{\ell(\mathbf{M})} \Rightarrow^* \varepsilon$$

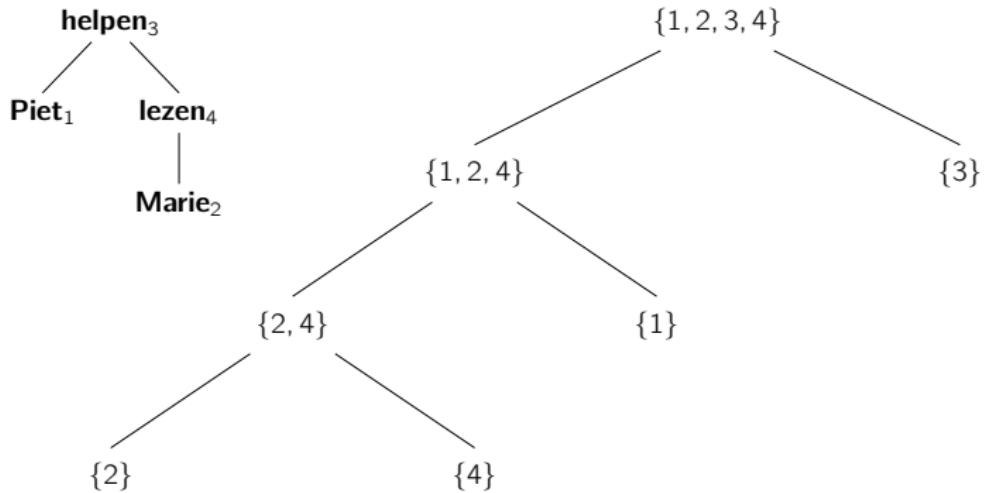
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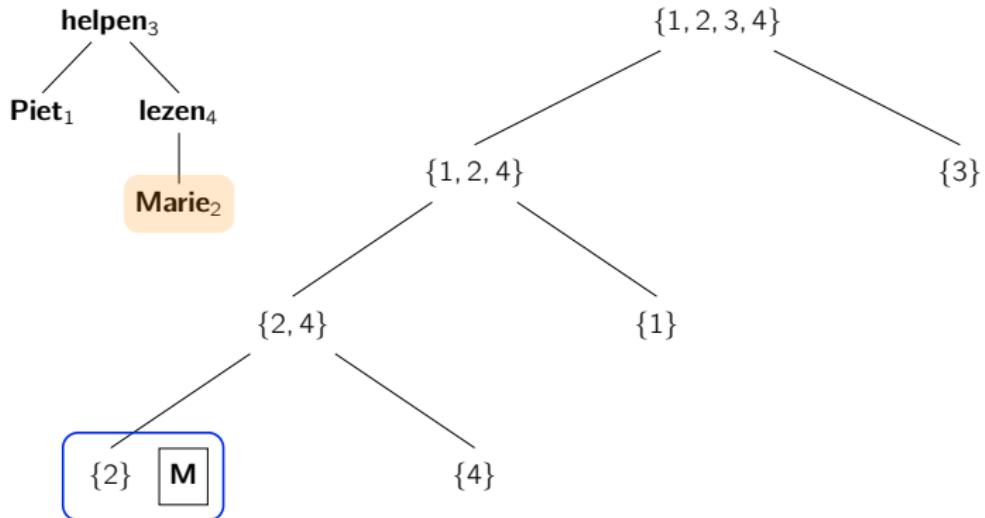
$$\{1, 2, 4\} \textcolor{red}{P} \textcolor{brown}{\ell(\mathbf{M})} \Rightarrow^* \varepsilon$$

$$\textcolor{teal}{\mathbf{M}} \{4\} \textcolor{teal}{\ell(\mathbf{M})} \Rightarrow^* \varepsilon$$

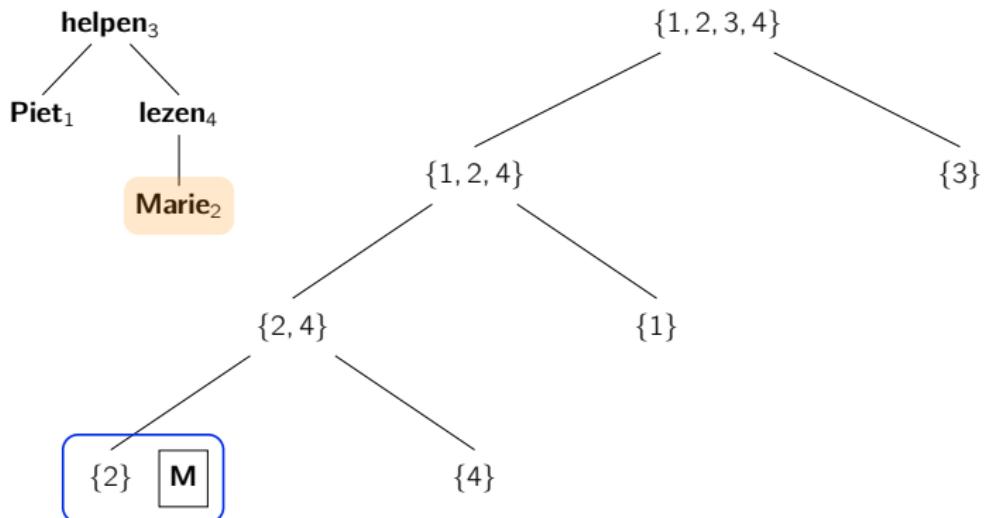
induction of sDCP G_2 :



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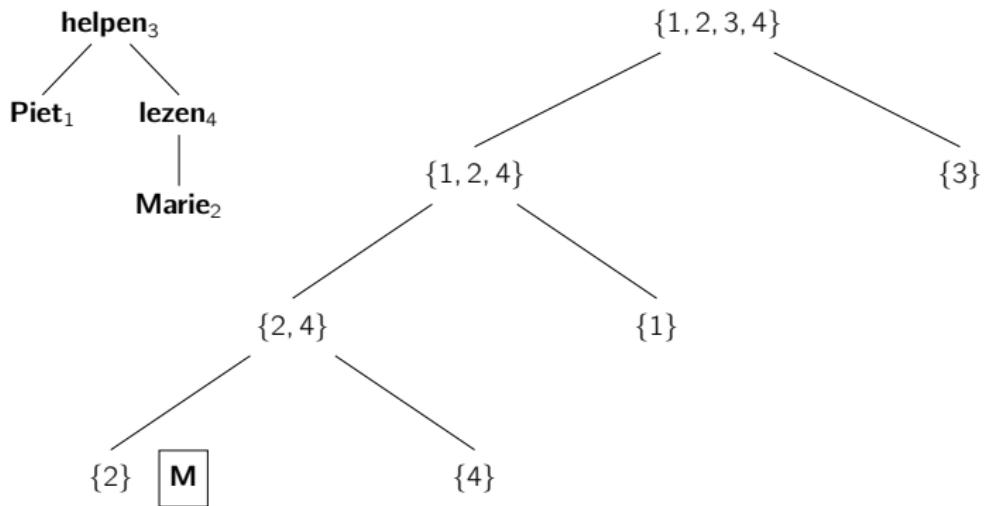


induction of sDCP G_2 :



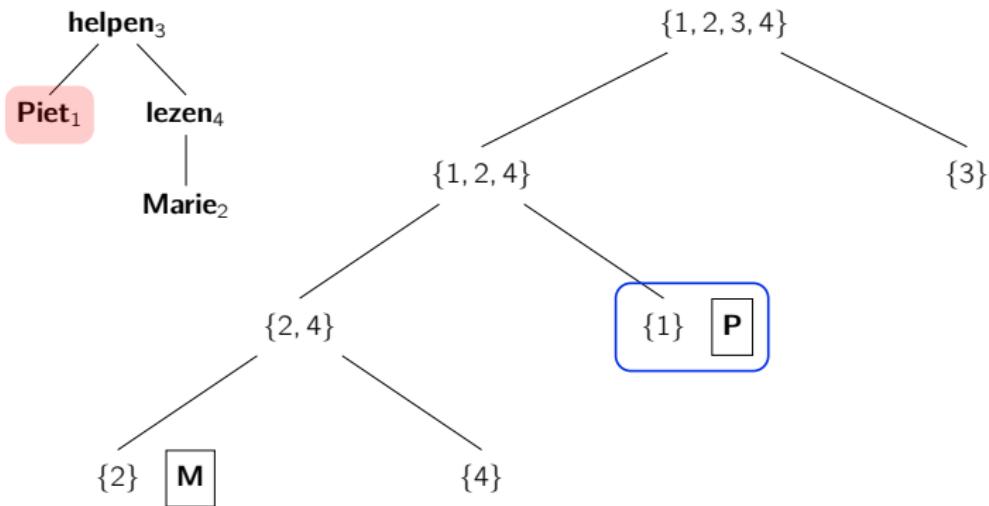
$\{2\}$ Marie $\rightarrow \varepsilon$

induction of sDCP G_2 :



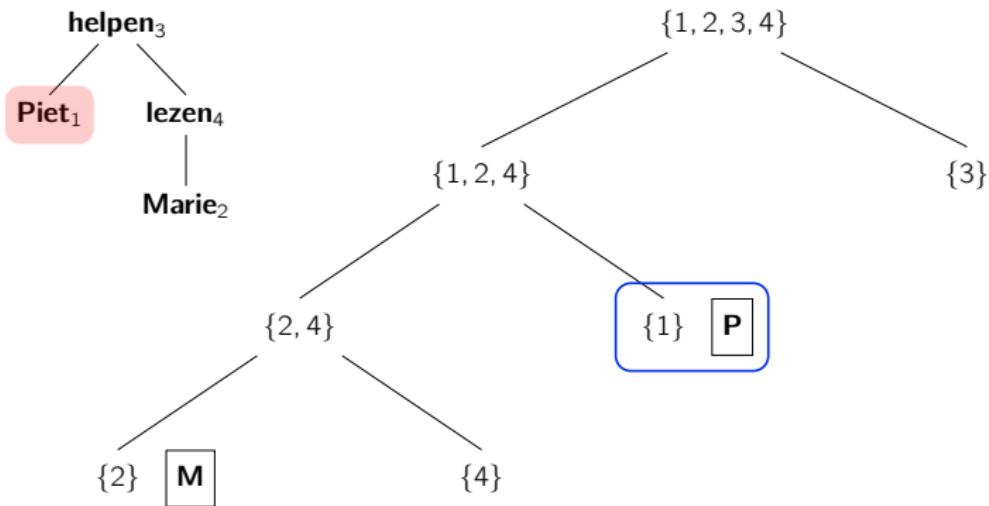
$\{2\}$ **Marie** $\rightarrow \varepsilon$

induction of sDCP G_2 :



$\{2\}$ Marie $\rightarrow \varepsilon$

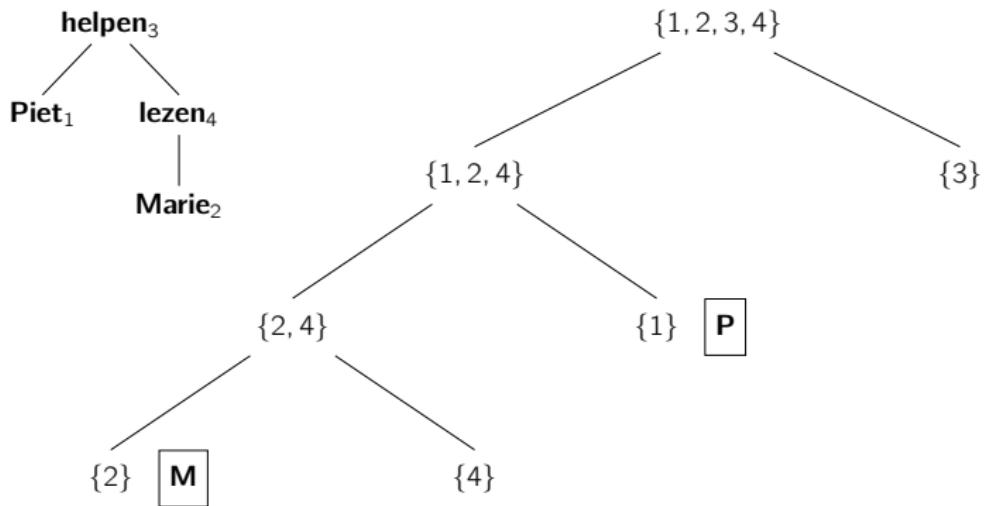
induction of sDCP G_2 :



{2} Marie → ε

{1} Piet → ε

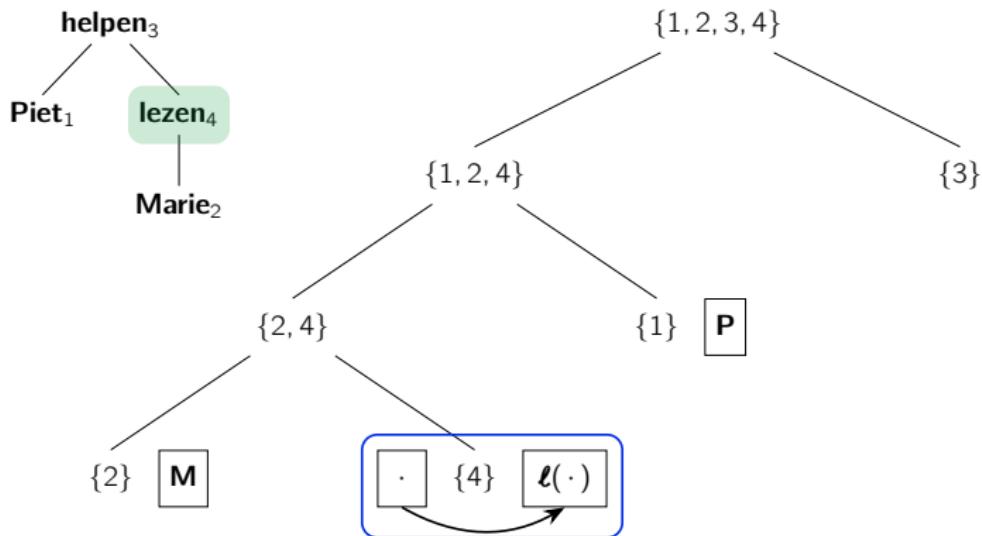
induction of sDCP G_2 :



$$\{2\} \boxed{\text{Marie}} \rightarrow \varepsilon$$

$$\{1\} \quad \boxed{\text{Piet}} \rightarrow \varepsilon$$

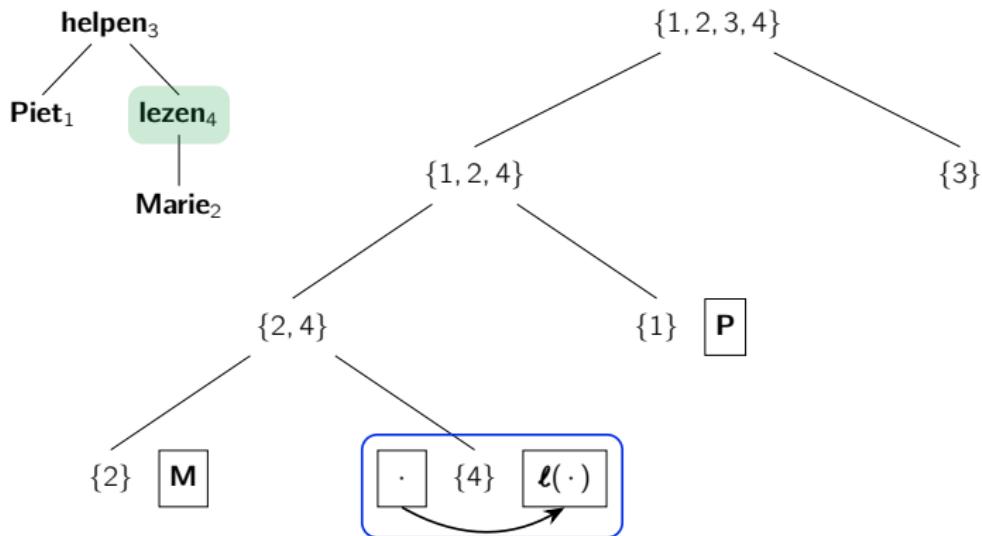
induction of sDCP G_2 :



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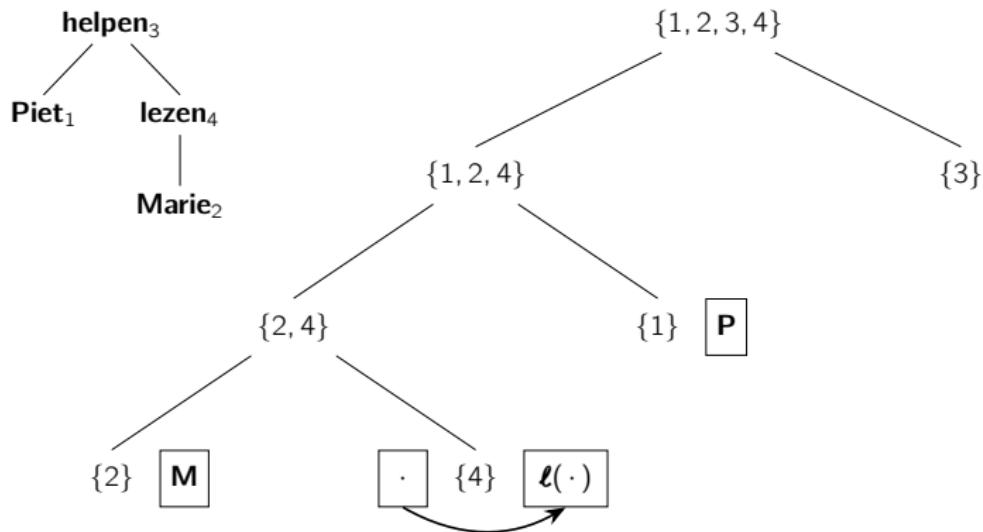


$$\{2\} \boxed{\text{Marie}} \rightarrow \varepsilon$$

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$$Z_1 \quad \{4\} \boxed{\begin{matrix} \text{lezen} \\ | \\ Z_1 \end{matrix}} \rightarrow \varepsilon$$

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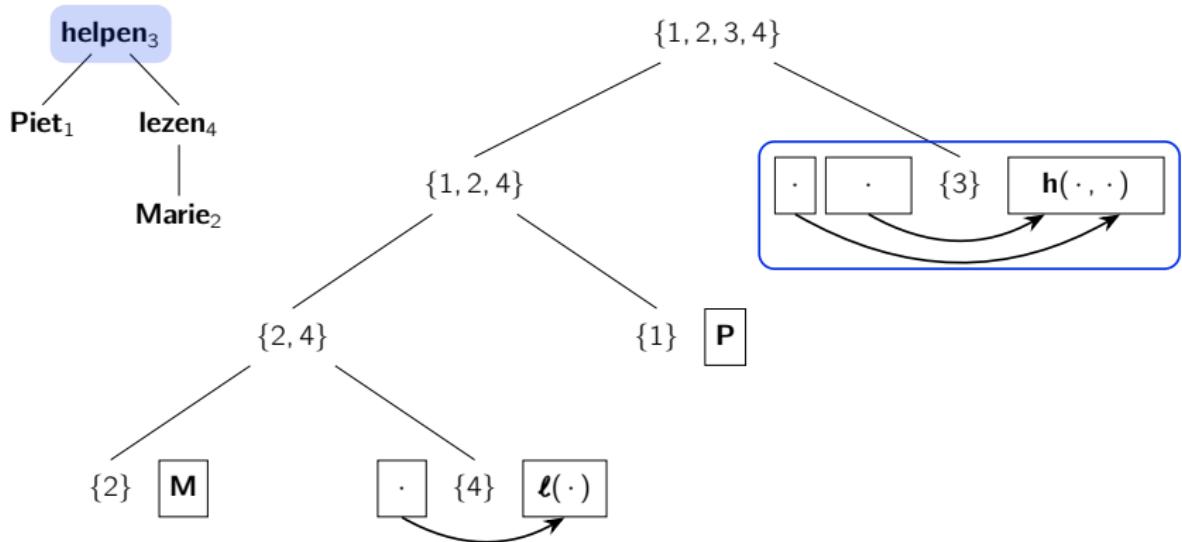


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$$\boxed{z_1} \quad \{4\} \quad \boxed{\text{lezen}} \quad | \quad z_1 \rightarrow \varepsilon$$

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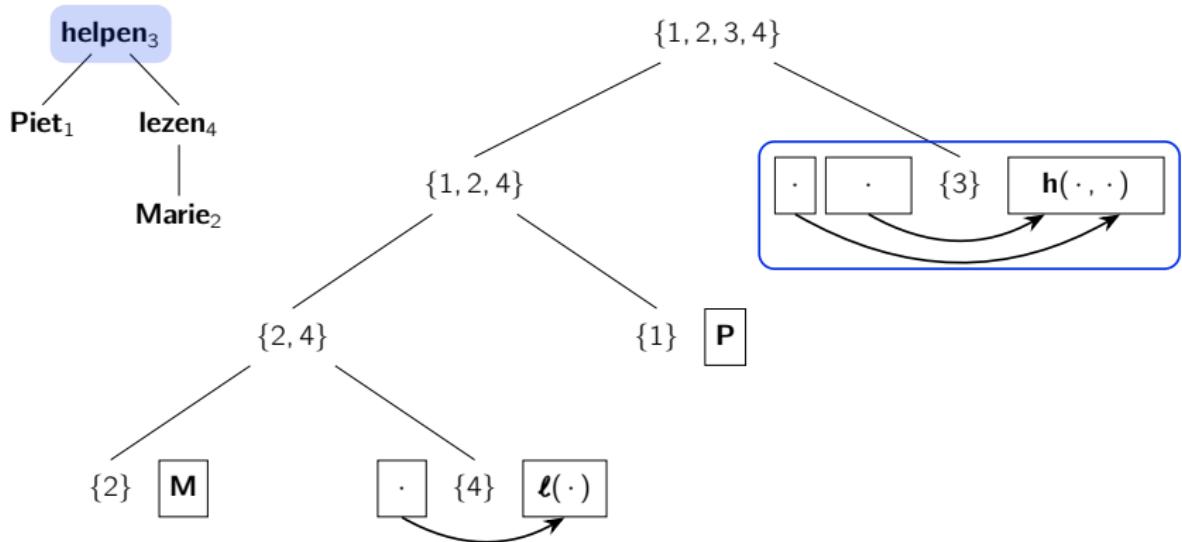


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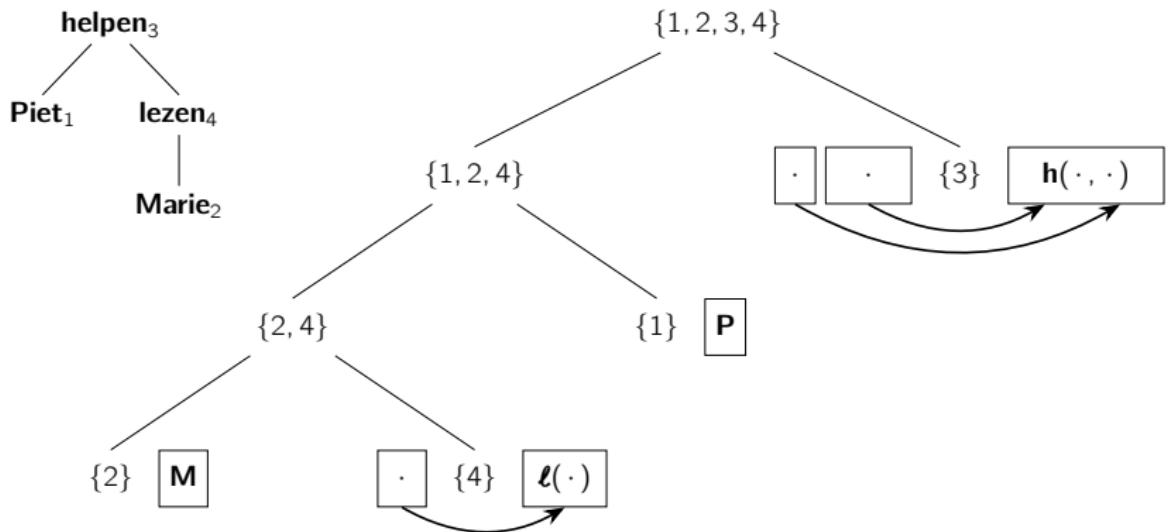
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$$\boxed{1} \quad \{4\} \quad \boxed{\text{lezen}} \quad | \quad Z_1 \rightarrow \varepsilon$$

$$Z_1 \quad Z_2 \quad \{3\} \quad \boxed{\text{helpen}} \quad / \quad \backslash \quad Z_1 \quad Z_2 \rightarrow \varepsilon$$

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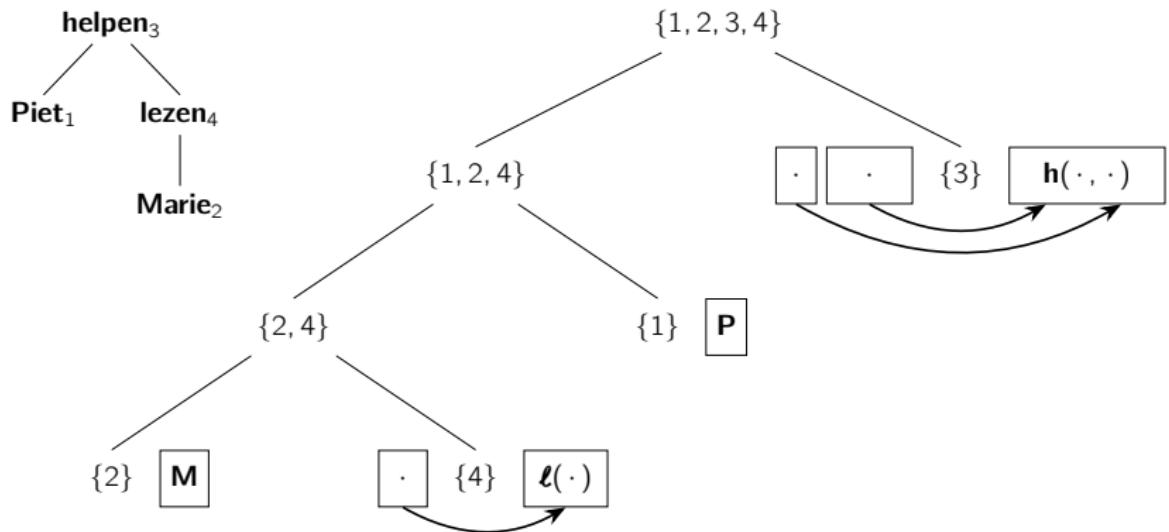
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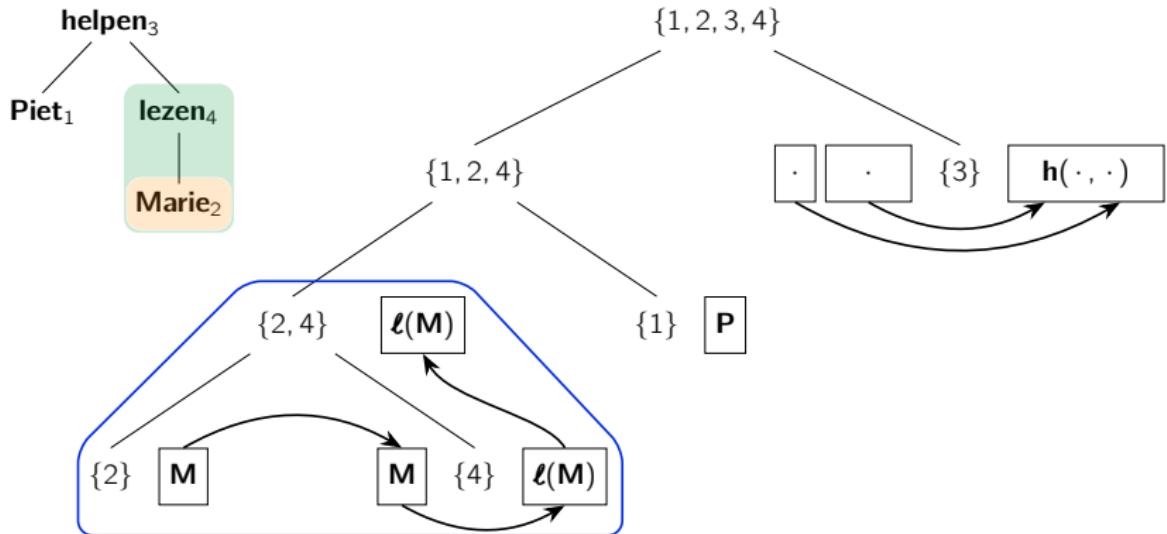
$$\boxed{\text{Z}_1} \quad \{4\} \quad \boxed{\text{lezen}} \quad | \quad Z_1 \rightarrow \varepsilon$$

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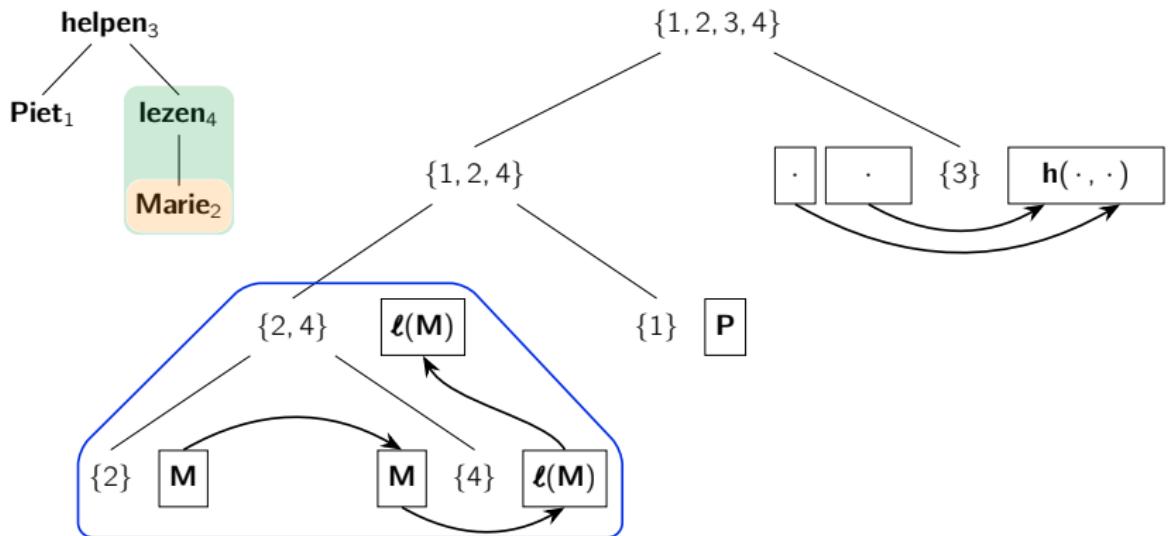
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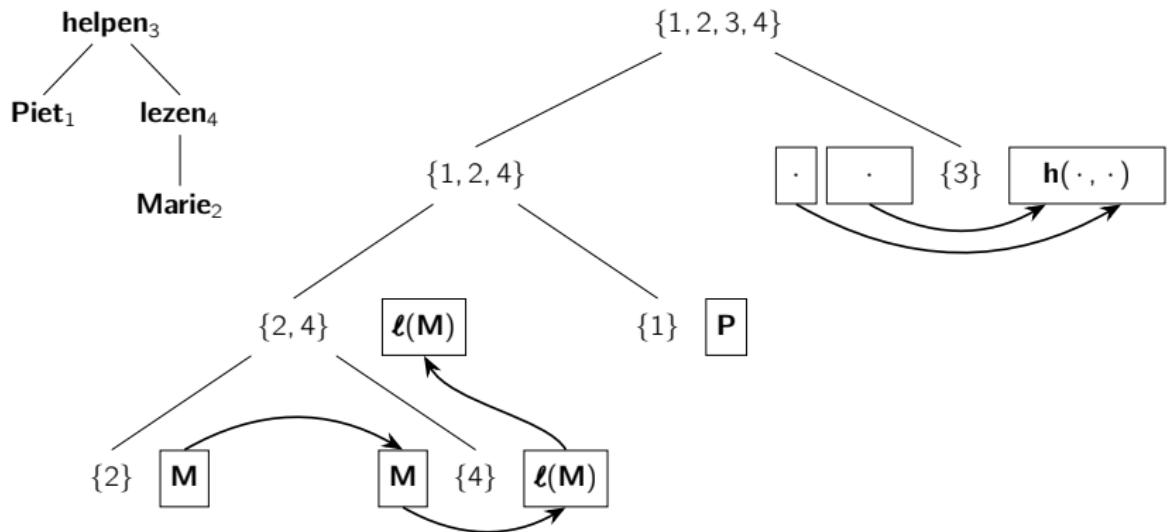


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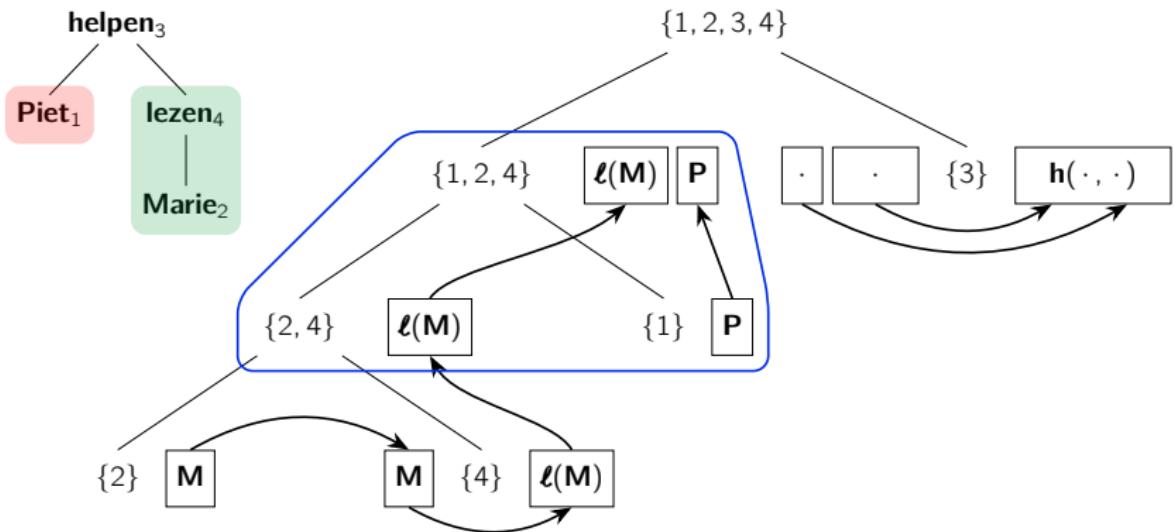
$$\{2, 4\} \boxed{z_2} \rightarrow \{2\} \boxed{z_1} \quad \boxed{z_1} \{4\} \boxed{z_2}$$

induction of sDCP G_2 :



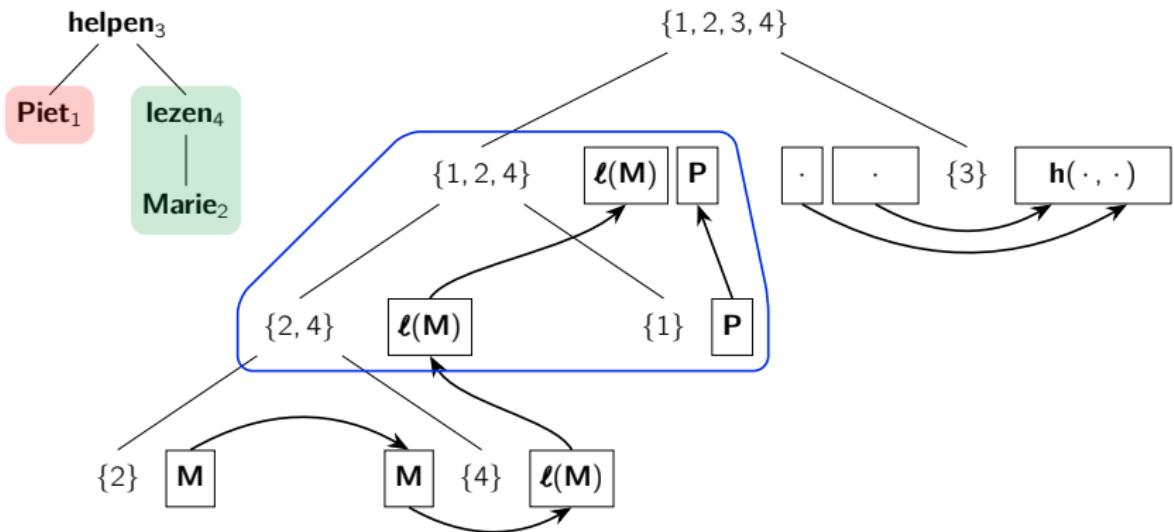
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induction of sDCP G_2 :



$$\{2, 4\} [z_2] \rightarrow \{2\} [z_1] \quad [z_1] \{4\} [z_2]$$

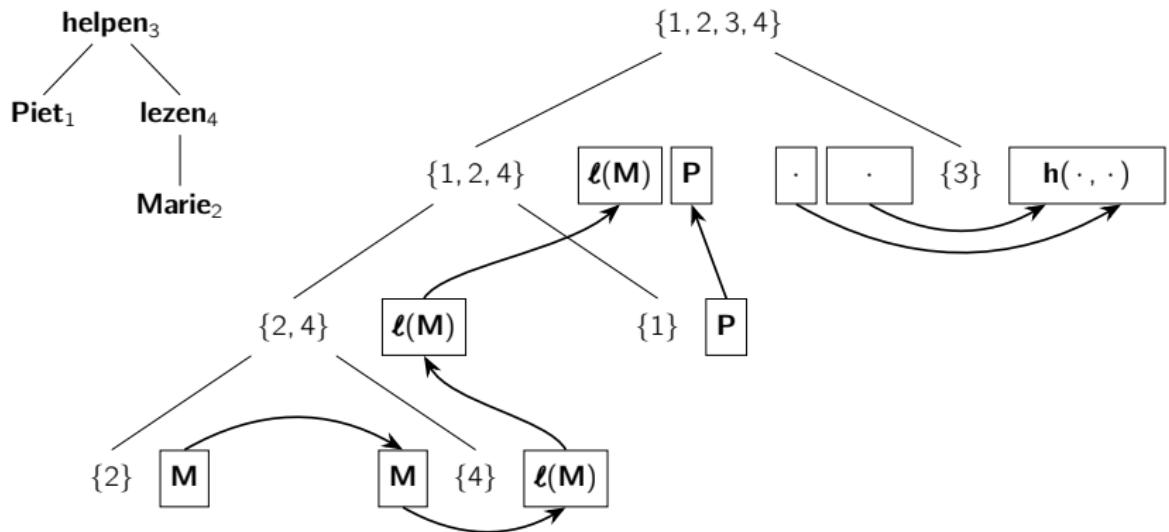
induction of sDCP G_2 :



$$\{2, 4\} [z_2] \rightarrow \{2\} [z_1] \quad [z_1] \{4\} [z_2]$$

$$\{1, 2, 4\} [z_1] [z_2] \rightarrow \{2, 4\} [z_1] \quad \{1\} [z_2]$$

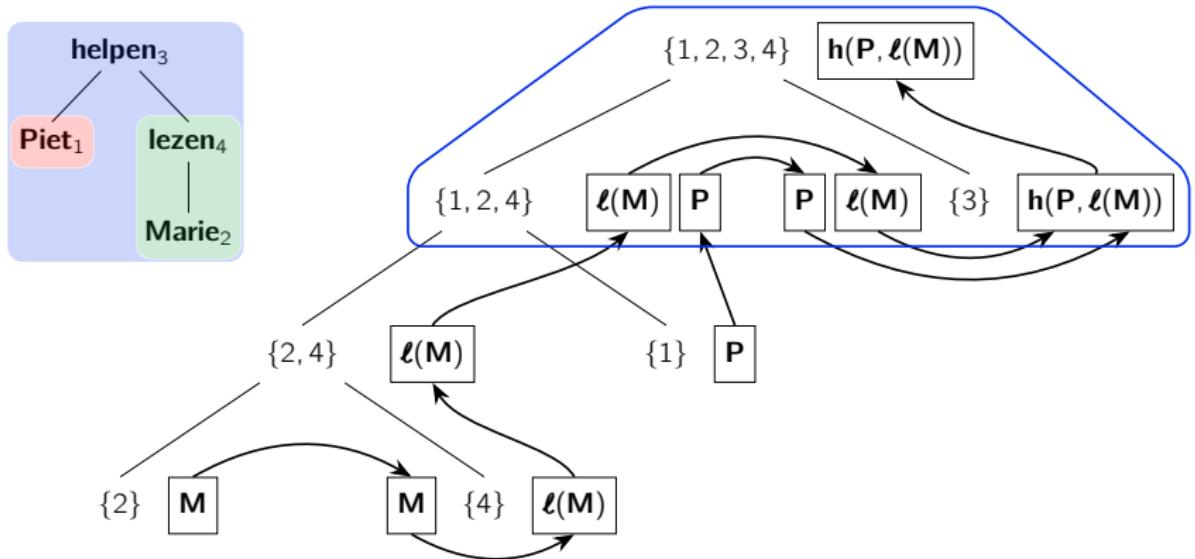
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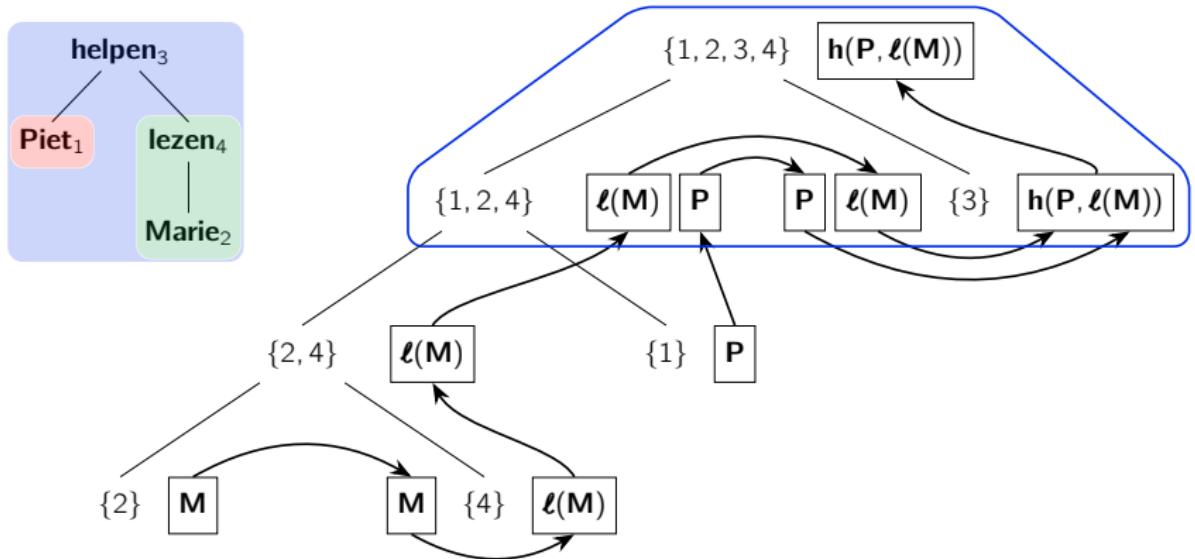
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$$\{1, 2, 4\} \boxed{z_1} \boxed{z_2} \rightarrow \{2, 4\} \boxed{z_1} \quad \{1\} \boxed{z_2}$$

$$\{1, 2, 3, 4\} \boxed{z_3} \rightarrow \{1, 2, 4\} \boxed{z_1} \boxed{z_2} \quad \boxed{z_2} \boxed{z_1} \{3\} \boxed{z_3}$$