Statistical Machine Translation of Natural Languages

Heiko Vogler
Technische Universität Dresden

Theorietag 2012
Prag, October 3, 2012

SMT = statistical machine translation

Weighted Tree Automata and Weighted Tree Transducers

can help in

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SMT = statistical machine translation
wta = weighted tree automata
wtt = weighted tree transducers

theory of wta and wtt

- determinization of wta
- minimization of wta
- (de-)composition of wtt
- input/output product
- Bar-Hillel, Shamir, Perles
  for wta and wsa
- characterization of wta
  (Büchi, Kleene, ...)
- variation of weight structure
  (semirings, fields, valuation mon.)
- ... [Fülöp, V. 09]

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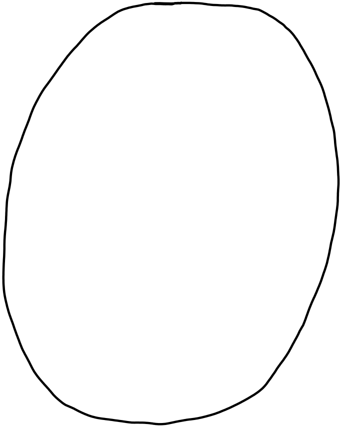

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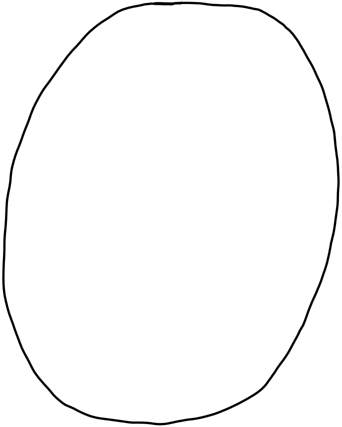

[Kevin Knight et al. 03-...]

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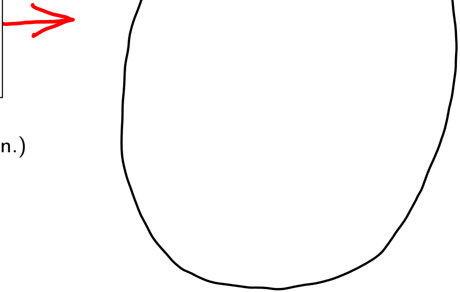

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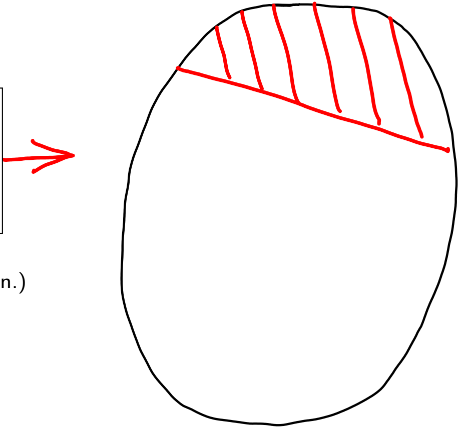

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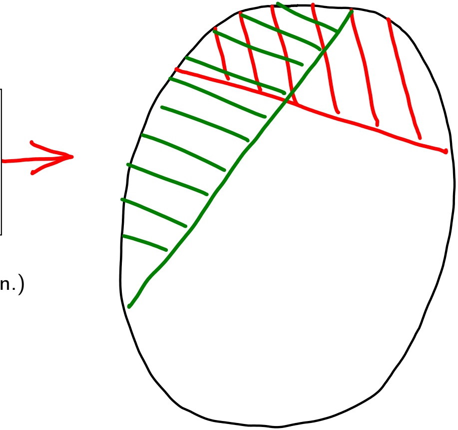

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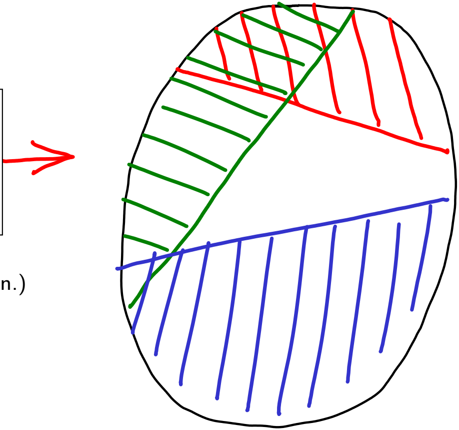

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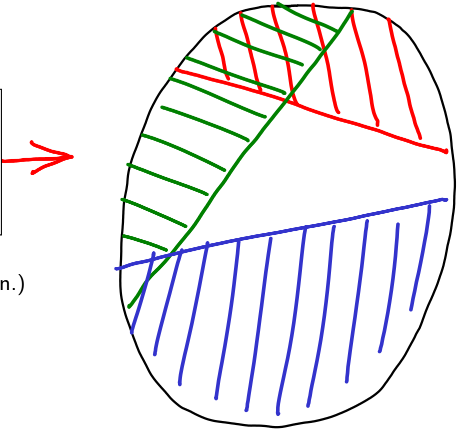

no survey on SMT!

outline of the talk:

- Statistical machine translation
  (modelling, training, evaluation)

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given:

- source language SL
- target language TL

find:

$$\text{translation} \quad h : \mathrm{SL} \to \mathrm{TL}$$

e.g.

SL = English $\quad$ s = I saw the man with the telescope

TL = German $\quad$ h(s) = Ich sah den Mann durch das Tel.

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machine translation $\rightsquigarrow$ statistical machine translation

[Lopez 08]: *"SMT treats the translation of natural languages as a machine learning problem."*

- assumptions $\longrightarrow$ modelling $\longrightarrow$ $\mathcal{H}$ hypothesis space

assumptions: mental work, experience, no data
hypothesis space: $\mathcal{H} \subseteq \{h \mid h : \mathrm{SL} \to \mathrm{TL}\}$

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- $\mathcal{H}$ and training data $\longrightarrow$ [training] $\longrightarrow$ $\hat{h} \in \mathcal{H}$

  [Lopez 08]: *"By examining many samples of human-produced translations, SMT algorithms automatically learn how to translate."*

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  goal/hope: $\hat{h}(s)$ is also "good" for $s \notin \{s_1, \ldots, s_n\}$

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- $\hat{h}$ and test data $\longrightarrow$ [evaluation] $\longrightarrow$ score

  BLEU (bilingual evaluation understanding), WER (word error rate), TER (translation error rate)

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NP	→	*NP PP*	#	0.5
NP	→	I	#	0.3
NP	→	the man	#	0.2
PP	→	with the tel.	#	1.0
VP	→	saw *NP PP*	#	0.4
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NP : noun phrase, VP: verb phrase, PP : prepositional phrase

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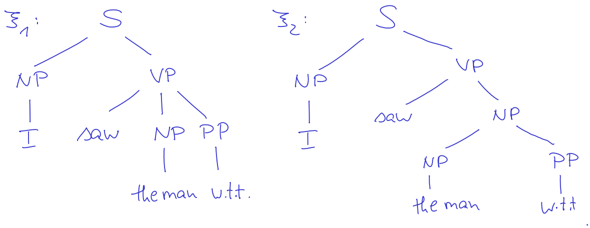

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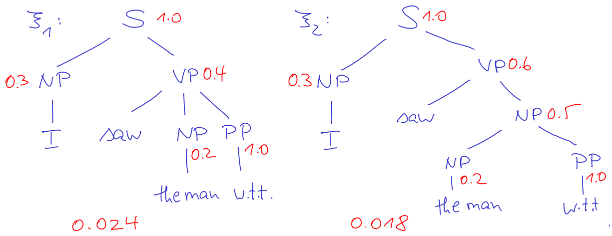

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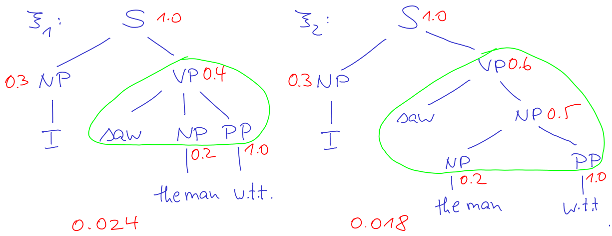

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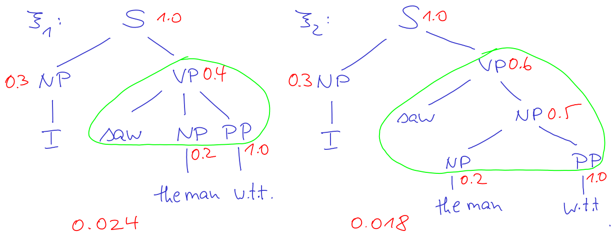

prob. cf grammar for English `gram3.txt`

(Stanford-parser `http://nlp.stanford.edu:8080/parser/index.jsp`)

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recognizable tree languages are closed under relabelings.

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weighted tree language: $L : T_{\Sigma} \to \mathbb{R}$

L is recognizable:

if there is a wta $\mathcal{A}$
which "recognizes" (computes) L

weighted tree automaton (wta) $\mathcal{A} = (Q, \Sigma, \delta, F)$

- Q finite set (states)
- Σ ranked alphabet (input symbols)
- $\delta = (\delta_\sigma \mid \sigma \in \Sigma)$ $\qquad \delta_\sigma : Q^k \times Q \to \mathbb{R}$

  $\delta_\sigma(q_1 \cdots q_k, q) \in \mathbb{R}$
- $F \subseteq Q$ (final states)

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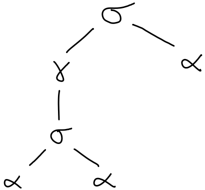

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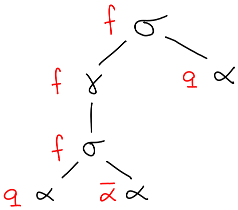

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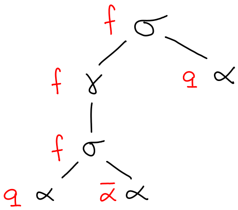

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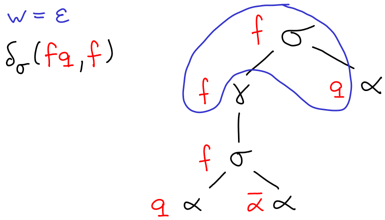

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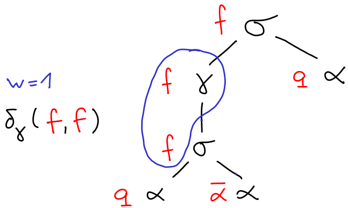

weighted tree automaton (wta) $\mathcal{A} = (Q, \Sigma, \delta, F)$

- Q finite set (states)
- Σ ranked alphabet (input symbols)
- $\delta = (\delta_\sigma \mid \sigma \in \Sigma)$ $\quad \delta_\sigma : Q^k \times Q \to \mathbb{R}$
  $\delta_\sigma(q_1 \cdots q_k, q) \in \mathbb{R}$
- $F \subseteq Q$ (final states)

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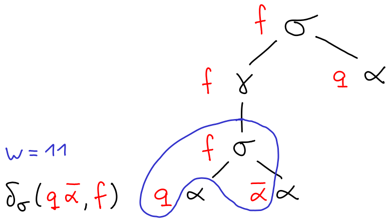

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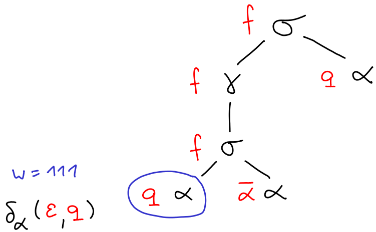

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weighted tree language recognized by $\mathcal{A}$:

$$L_\mathcal{A} : T_\Sigma \to \mathbb{R}, \quad L_\mathcal{A}(\xi) = \max_{r \in R_\mathcal{A}(\xi)} \mathrm{wt}(r)$$

second assumption:

translation from SL and TL is specified by a weighted tree transducer

[Yamada, Knight 01] translation from English to Japanese

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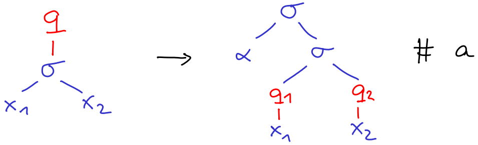
$$\rho : \; q(\sigma(x_1, \ldots, x_k)) \to \xi[q_1(x_1), \ldots, q_k(x_k)] \; \# \; a$$

$q, q_1, \ldots, q_k \in Q$, $\sigma \in \Sigma$, $\xi \in T_\Sigma(X_k)$
ξ linear, nondeleting in $x_1, \ldots, x_k$
$a \in \mathbb{R}$



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weighted tree transformation computed by $\mathcal{M}$:

$$\tau_{\mathcal{M}} : T_\Sigma \times T_\Sigma \rightarrow \mathbb{R}, \quad \tau_{\mathcal{M}}(\xi_1, \xi_2) = \max_{\substack{d \in D_{\mathcal{M}}: \\ \pi(d) = (\xi_1, \xi_2)}} \mathrm{wt}(d)$$

$D_{\mathcal{M}}$: set of all derivations

modelling with wtt and wta:

- model for translation from SL to TL: wtt $\mathcal{M}$
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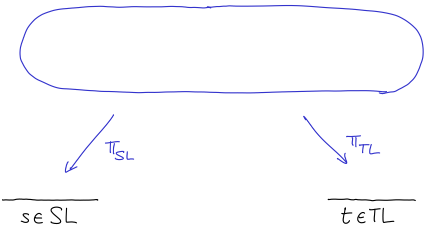

correspondence structure:
[Liang et al. 06]

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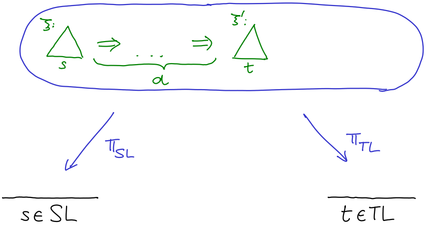

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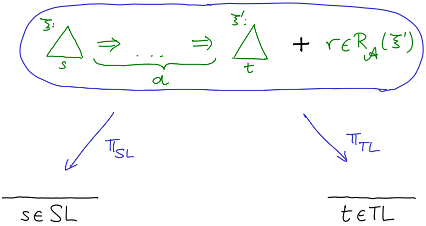

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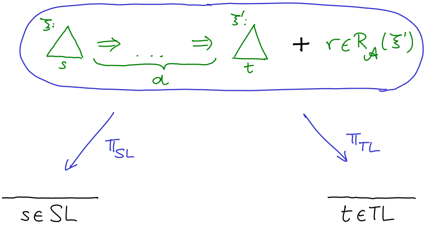

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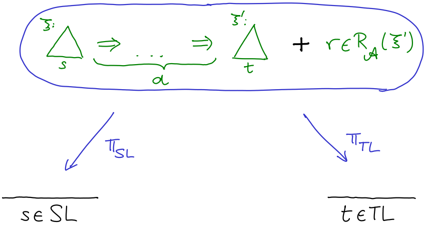

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hypothesis space: $\mathcal{H} = \{h_{\mathcal{M},\mathcal{A}} \mid \mathrm{wtt}\ \mathcal{M}, \mathrm{wta}\ \mathcal{A}\}$

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outline of the talk:

- Statistical machine translation
  (modelling, training, evaluation)
- Modelling with wta and wtt
- Using output product to improve modelling
- Using Bar-Hillel, Shamir, Perles and input product
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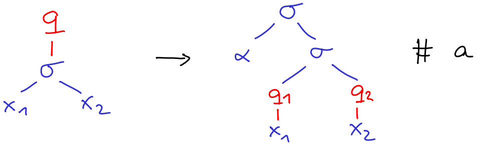

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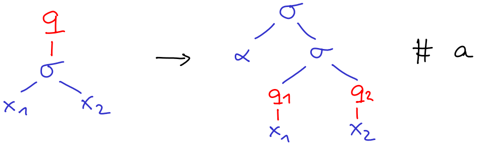

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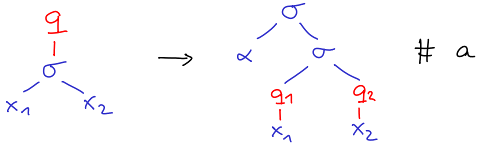

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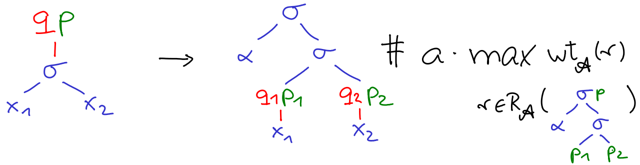
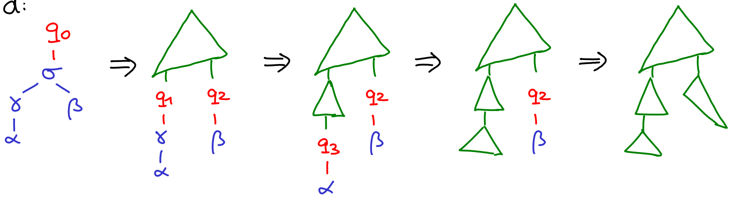


d:



q_0
σ
γ
β
α
$\Rightarrow$
q_1
q_2
γ
β
α
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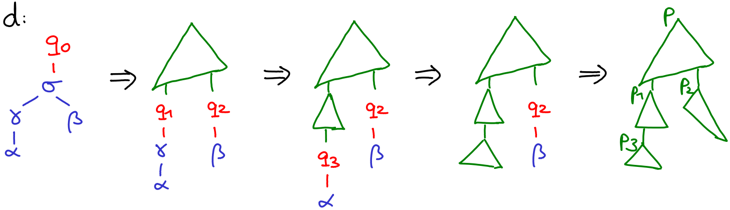
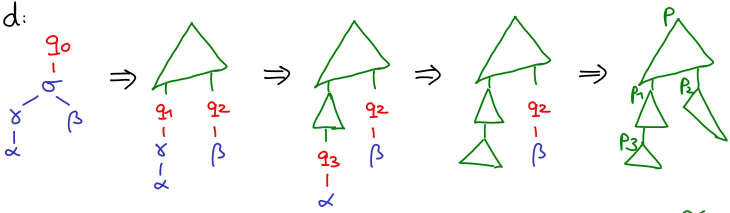


+ partial run $\tilde{p}$



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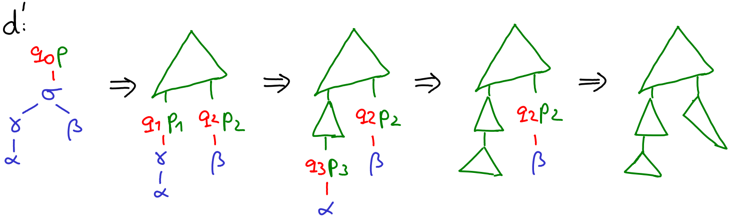

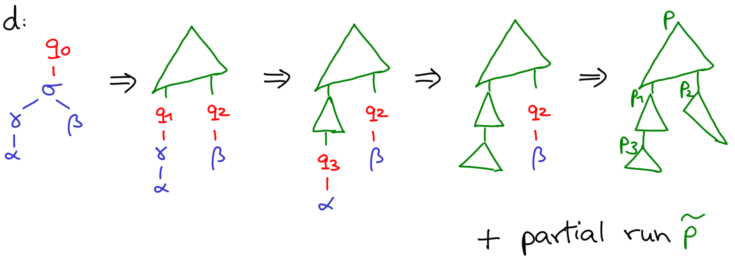

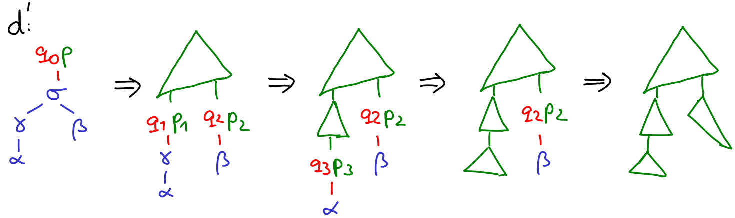

$\varphi : D_{\mathcal{M}'} \to \{(d, \tilde{p}) \mid d \in D_{\mathcal{M}}, \tilde{p} \in R_{\mathcal{A}}^{\text{partial}}(d)\}$ bijection

$\mathrm{wt}(d') = \mathrm{wt}(d) \cdot \max_{r \in \mathrm{completion}(\tilde{p})} \mathrm{wt}(r)$

recall:

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Theorem [Büchse, Nederhof, V. 11]:
Let $\mathcal{M}$ synchronized tree-adjoining grammar (STAG)
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outline of the talk:

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given: $h_{\mathcal{M},\mathcal{A}} : \mathrm{SL} \to \mathrm{TL}$ and $s \in \mathrm{SL}$

compute: $h_{\mathcal{M},\mathcal{A}}(s) = \pi_{\mathrm{TL}}\left(\mathrm{argmax}_{\substack{d \in D_{\mathcal{M} \triangleright \mathcal{A}}: \\ \pi_{\mathrm{SL}}(d) = s}} \ \mathrm{wt}(d)\right)$

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construct: wta $\mathcal{A}_s$ such that for every $\xi \in T_\Sigma$

$$L_{\mathcal{A}_s}(\xi) = \begin{cases} 1 & \text{if } \mathrm{yield}(\xi) = s \\ 0 & \text{otherwise} \end{cases}$$

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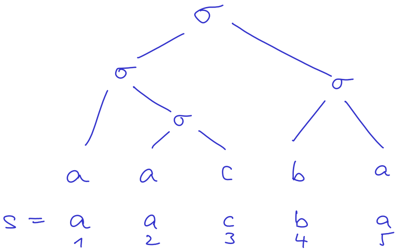

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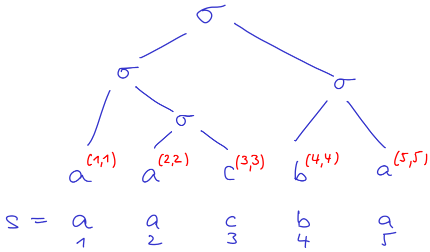

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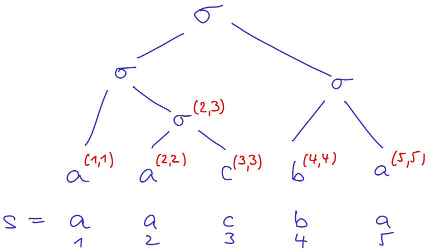

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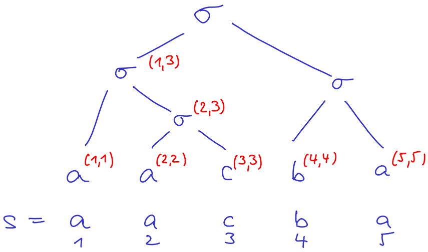

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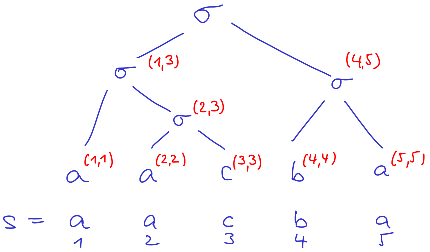

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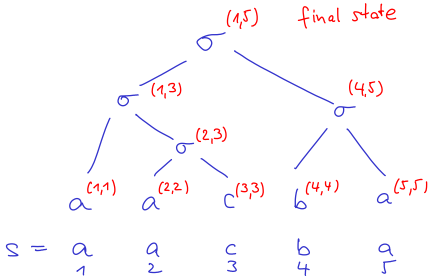

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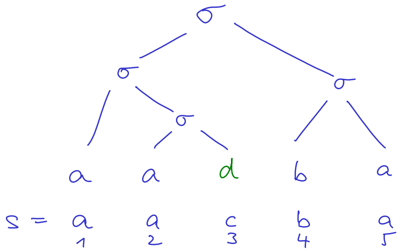

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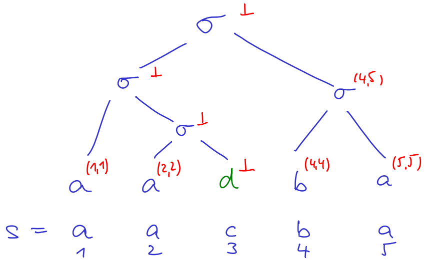

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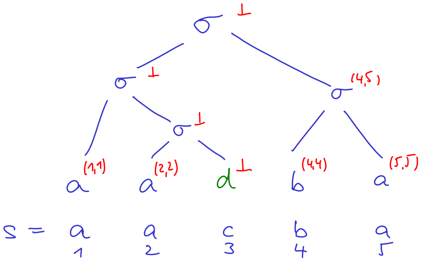

> Theorem [Bar-Hillel, Shamir, Perles 61]
> The class of cf languages is closed under intersection with regular languages.

recall:

$h_{\mathcal{M},\mathcal{A}} : \mathrm{SL} \to \mathrm{TL}$

$$s \quad \mapsto \quad \pi_{\mathrm{TL}}\left(\operatorname{argmax}_{\substack{d \in D_{\mathcal{M} \triangleright \mathcal{A}}: \\ \pi_{\mathrm{SL}}(d) = s}} \mathrm{wt}(d)\right)$$

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$$= \pi_{\mathrm{TL}} \left(\mathrm{argmax}_{\substack{(d,r): \\ d \in D_{\mathcal{M} \triangleright \mathcal{A}} \\ r \in R_{\mathcal{A}_s}(\mathrm{first}(d))}} \mathrm{wt}(r) \cdot \mathrm{wt}(d) \right)$$

$h_{\mathcal{M},\mathcal{A}} : \mathrm{SL} \to \mathrm{TL}$

$$
\begin{aligned}
s &\mapsto \pi_{\mathrm{TL}}\left(\operatorname{argmax}_{\substack{d \in D_{\mathcal{M} \triangleright \mathcal{A}}:\\ \pi_{\mathrm{SL}}(d)=s}} \mathrm{wt}(d)\right) \\
&= \pi_{\mathrm{TL}}\left(\operatorname{argmax}_{\substack{(d,r):\\ d \in D_{\mathcal{M} \triangleright \mathcal{A}}\\ r \in R_{\mathcal{A}_s}(\mathrm{first}(d))}} \mathrm{wt}(r) \cdot \mathrm{wt}(d)\right) \\
&= \pi_{\mathrm{TL}}\left(\operatorname{argmax}_{d \in D_{\mathcal{A}_s \triangleleft (\mathcal{M} \triangleright \mathcal{A})}} \mathrm{wt}(d)\right)
\end{aligned}
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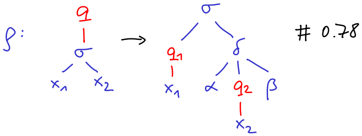

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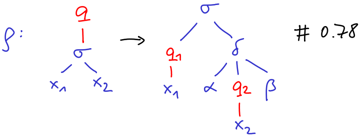
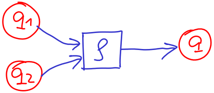
hypergraph $G(\mathcal{M}')$





$g_\rho : [0,1] \times [0,1] \to [0,1]$

$(n_1, n_2) \mapsto n_1 \cdot n_2 \cdot 0.78$

- $g_\rho(n_1, n_2) \leq \min(n_1, n_2)$
- g_ρ monotone

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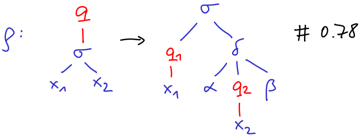

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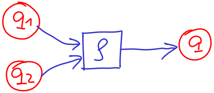

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hypergraph $G(\mathcal{M}')$ $\longrightarrow$ **Knuth's algorithm** $\longrightarrow$ $\operatorname{argmax}_{d \in D_{\mathcal{M}'}} \mathrm{wt}_{\mathcal{M}'}(d)$

[Knuth 77] A generalization of Dijkstra's shortest path algorithm

$\mathcal{O}(|E| \cdot \log |V|)$

summary:

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- model for translation from SL to TL: wtt $\mathcal{M}$

summary:

- model for translation from SL to TL: wtt $\mathcal{M}$
- model for TL: wta $\mathcal{A}$

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- model for translation from SL to TL: wtt $\mathcal{M}$
- model for TL: wta $\mathcal{A}$
- sentence s of SL

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noisy channel:

$$h_{\mathcal{M},\mathcal{A}}(s) \quad = \quad \pi_{\mathrm{TL}}\left(\operatorname{argmax}_{\substack{(d,r)\in Y:\\ \pi_{\mathrm{SL}}(d,r)=s}} \mathrm{wt}(d)\cdot\mathrm{wt}(r)\right)$$

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weighted tree automata and weighted tree transducers

can help in modelling and decoding of

statistical machine translation of natural languages

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but: SMT is an engineering task!

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conceptually

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References:

- [Baker 79] Composition of top-down and bottom-up tree transductions
- [Bar-Hillel, Shamir, Perles 61] On formal properties of simple phrase structure grammars
- [Büchse, Nederhof, V. 11] Tree Parsing with Synchronous Tree-Adjoining Grammars
- [Engelfriet, Fülöp, V. 02] Bottom-up and Top-down Tree Series Transformations
- [Fülöp, V. 09] Weighted tree automata and tree transducers
- [Knight et al. 03-...] ...
- [Knuth 77] A generalization of Dijkstra's algorithm
- [Lopez 08] Statistical Machine Translation
- [Liang, Bouchard-Côté, Klein, Taskar 06] An End-to-End Discriminative Approach to Machine Translation
- [Maletti 06] Compositions of Tree Series Transformations
- [Maletti, Satta 09] Parsing Algorithms based on Tree Automata
- [Nederhof, V. 12] Synchronous Context-Free Tree Grammars
- [Thatcher 67] Characterizing derivation trees of context-free grammars through a generalization of finite automata theory
- [Yamada, Knight 01] A syntax-based statistical translation model

[Knight et al. 03-...]

- [Charniak, Knight, Yamada 03] Syntax-based language models for statistical machine translation
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Thank you!