Recognizable Weighted Tree Languages - A Survey

Heiko Vogler
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ATANLP 2012
Avignon, April 24, 2012

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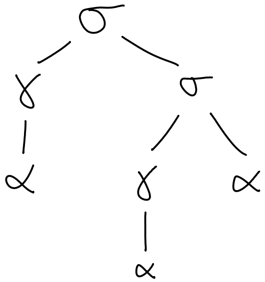
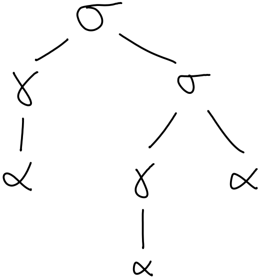
outline of my talk:

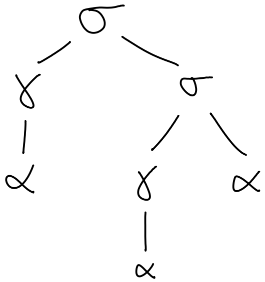
- from finite-state tree automata to weighted tree automata
- ...



σ
γ
σ
α
γ
α
α



Σ: ranked alphabet $\qquad \Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$



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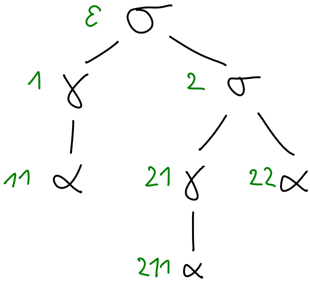
T_Σ: set of trees over Σ $\qquad \sigma(\gamma(\alpha), \sigma(\gamma(\alpha), \alpha))$



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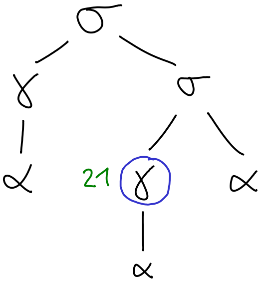
T_Σ: set of trees over Σ $\sigma(\gamma(\alpha), \sigma(\gamma(\alpha), \alpha))$

$\mathrm{pos}(\xi)$: set of positions of ξ $\mathrm{pos}(\xi) = \{\varepsilon, 1, 11, 2, 21, 211, 22\}$



Σ: ranked alphabet	$\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$
T_Σ: set of trees over Σ	$\sigma(\gamma(\alpha), \sigma(\gamma(\alpha), \alpha))$
$\mathrm{pos}(\xi)$: set of positions of ξ	$\mathrm{pos}(\xi) = \{\varepsilon, 1, 11, 2, 21, 211, 22\}$
$\xi(w)$: label of ξ at position $w \in \mathrm{pos}(\xi)$	$\xi(21) = \gamma$

tree language $L \subseteq T_\Sigma$

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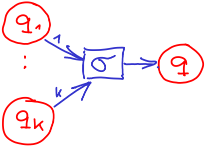
transition

$(q_1 \cdots q_k, \sigma, q) \rightsquigarrow$

hyperedge



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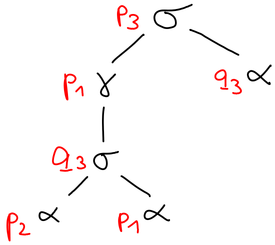

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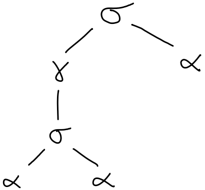

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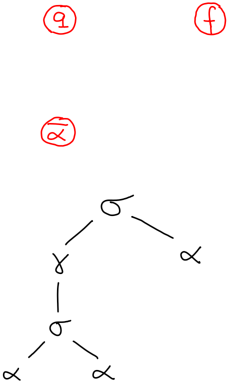

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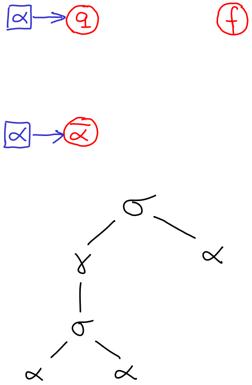

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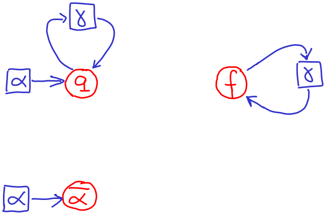

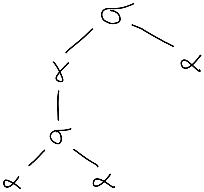

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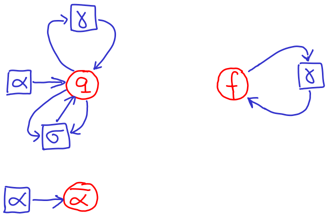

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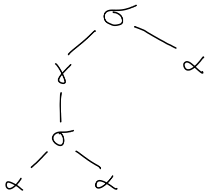

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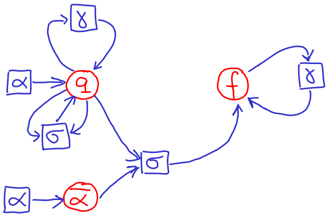

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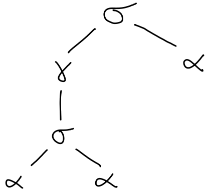

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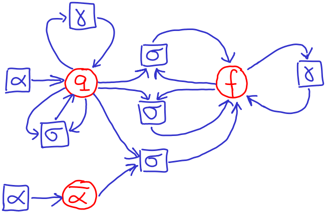

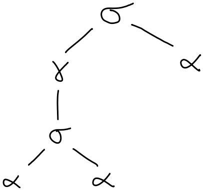

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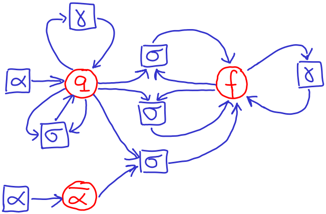

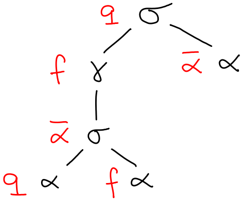

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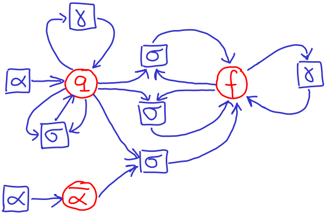

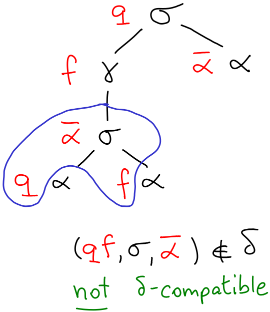

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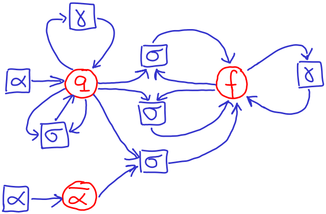

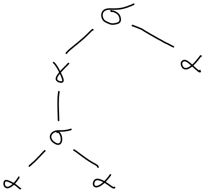

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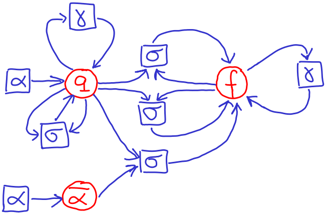

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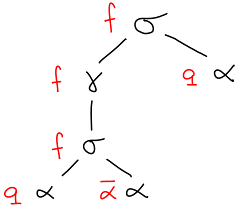

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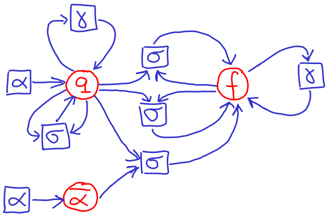

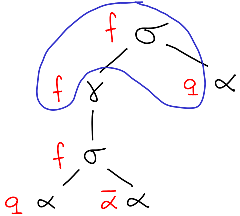

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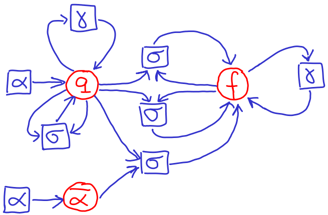

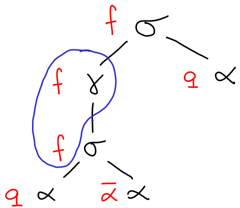

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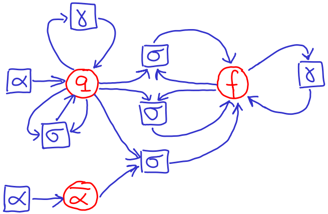

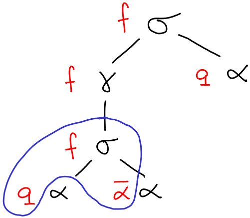

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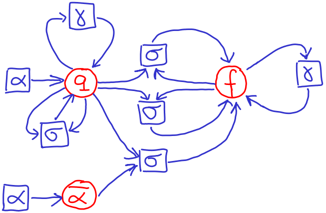

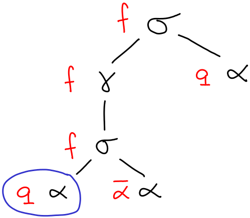

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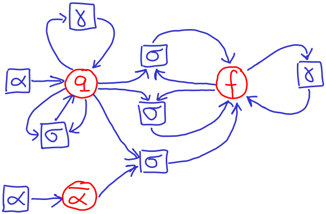

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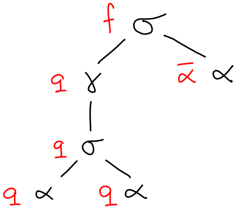

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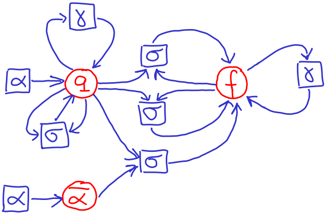
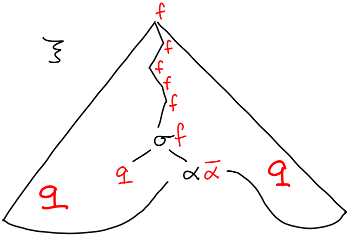
$\delta \subseteq \bigcup_{k \in \mathbb{N}} Q^k \times \Sigma^{(k)} \times Q$
$(q_1 \dots q_k, \sigma, q) \in \delta$
$F \subseteq Q$



tree lang. recognized by $\mathcal{A}$:
$\xi \in L_{\mathcal{A}}$ iff

there is $\kappa : \mathrm{pos}(\xi) \to Q$ s.t.
κ is δ-comp. and $\kappa(\varepsilon) \in F$



tree lang. L is *recognizable*
if $\exists$ fta $\mathcal{A}$ with $L = L_{\mathcal{A}}$

tree language $L \subseteq T_\Sigma$

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$\mathbb{B}$-wta $\mathcal{A} = (Q, \Sigma, \delta, F)$
$\delta : \bigcup_{k \in \mathbb{N}} Q^k \times \Sigma^{(k)} \times Q \to \mathbb{B}$
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$F : Q \to \mathbb{B}$

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$$L_\mathcal{A}(\xi) = \bigvee_{\kappa : \mathrm{pos}(\xi) \to Q} \Big(\mathrm{comp}(\kappa) \wedge F(\kappa(\varepsilon))\Big)$$

$\mathrm{comp}(\kappa) =$
$\delta(\varepsilon, \alpha, q) \wedge \delta(\varepsilon, \alpha, \overline{\alpha}) \wedge \dots$

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$(\mathbb{B}, \vee, \wedge) \rightsquigarrow (\mathbb{N}, +, \cdot)$

δ-compatible $\rightsquigarrow$ wt

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δ-compatible $\rightsquigarrow$ wt

$\mathbb{N}$-weighted tree lang. recogn. by $\mathcal{A}$

$$L_{\mathcal{A}}(\xi) = \sum_{\kappa : \mathrm{pos}(\xi) \to Q} \Big(\mathrm{wt}(\kappa) \cdot F(\kappa(\varepsilon))\Big)$$

$\mathrm{wt}(\kappa) =$
$\delta(\varepsilon, \alpha, q) \cdot \delta(\varepsilon, \alpha, \overline{\alpha}) \cdot \ldots$

$\mathbb{N}$-weighted tree lang. L is $\mathbb{N}$-*recogn.* if $\exists$ $\mathbb{N}$-wta $\mathcal{A}$ with $L = L_{\mathcal{A}}$

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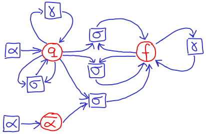

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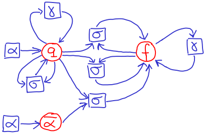
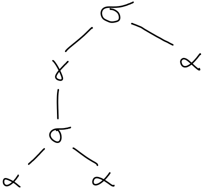


$$L_{\mathcal{A}}(\xi) = \sum_{\kappa:\text{pos}(\xi)\to Q} \Big(\text{wt}(\kappa) \cdot F(\kappa(\varepsilon))\Big)$$



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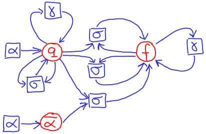
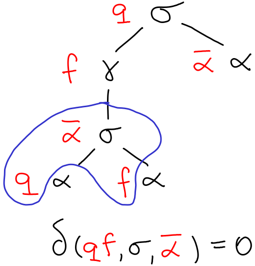


$L_{\mathcal{A}}(\xi) = \mathrm{wt}(\kappa_1) \cdot F(q) + \ldots$



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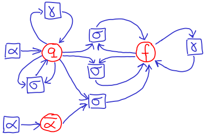
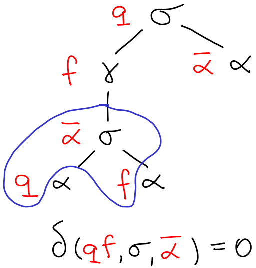


$L_{\mathcal{A}}(\xi) = \quad 0 + \dots$



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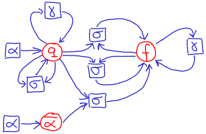
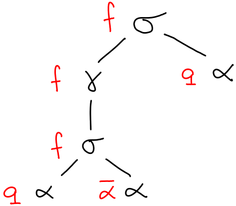


$L_{\mathcal{A}}(\xi) = \quad 0 + \mathrm{wt}(\kappa_2) \cdot F(f) + \ldots$



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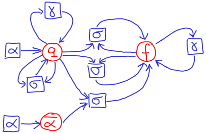
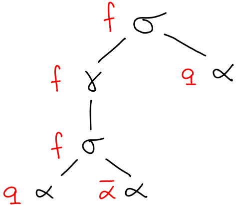


$L_{\mathcal{A}}(\xi) = \quad 0 + 1 + \ldots$



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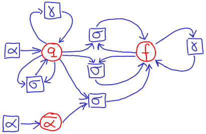
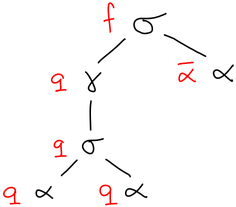


$L_{\mathcal{A}}(\xi) = \quad 0 + 1 + \mathrm{wt}(\kappa_3) \cdot F(f) + ..$



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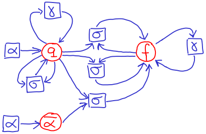
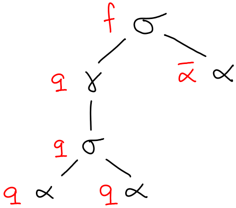


$L_{\mathcal{A}}(\xi) = \quad 0 + 1 + 1 + \ldots$



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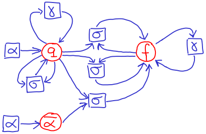
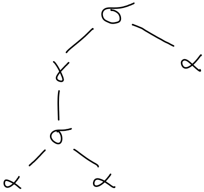


$L_{\mathcal{A}}(\xi) = \quad 0 + 1 + 1 + 0 + \ldots + 0 = 2$



$L_{\mathcal{A}}(\xi) = \#$ occurrences of $\sigma(., \alpha)$ in ξ

$\mathbb{N}$-weighted tree lang. $L : T_\Sigma \to \mathbb{N}$

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$\mathbb{N}$-weighted tree lang. recogn. by $\mathcal{A}$

$$L_\mathcal{A}(\xi) = \sum_{\kappa : \mathrm{pos}(\xi) \to Q} \Big(\mathrm{wt}(\kappa) \cdot F(\kappa(\varepsilon)) \Big)$$

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$(\mathbb{N}, +, \cdot) \rightsquigarrow (\mathbb{S}, \oplus, \odot)$

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$\mathbb{S}$-weighted tree lang. recogn. by $\mathcal{A}$

$$L_{\mathcal{A}}(\xi) = \bigoplus_{\kappa : \mathrm{pos}(\xi) \to Q} \Big(\mathrm{wt}(\kappa) \odot F(\kappa(\varepsilon)) \Big)$$

calculation of $\odot$
depth-first left-to-right

$\mathrm{wt}(\kappa) =$
$\delta(\varepsilon, \alpha, q) \odot \delta(\varepsilon, \alpha, \overline{\alpha}) \odot \ldots$

$\mathbb{S}$-weighted tree lang. L is $\mathbb{S}$-*recogn.*
if $\exists$ $\mathbb{S}$-wta $\mathcal{A}$ with $L = L_{\mathcal{A}}$

$(\mathbb{S}, \oplus, \odot, \mathbf{0}, \mathbf{1})$ semiring

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- $\mathbf{1}$ is neutral w.r.t. $\odot$ (i.e., $a \odot \mathbf{1} = \mathbf{1} \odot a = a$),
- $\mathbf{0}$ is absorbing w.r.t. $\odot$ (i.e., $a \odot \mathbf{0} = \mathbf{0} \odot a = \mathbf{0}$),
- $\odot$ distributes over $\oplus$

  (i.e., $a \odot (b \oplus c) = (a \odot b) \oplus (a \odot c)$
  $(a \odot b) \oplus c = (a \odot c) \oplus (b \odot c)$).

$(\mathbb{S}, \oplus, \odot, \mathbf{0}, \mathbf{1})$ semiring

$(\mathbb{S}, \oplus, \odot, \mathbf{0}, \mathbf{1})$ semiring

Examples of semirings (s.r.):

- $(\mathbb{B}, \vee, \wedge, \mathit{ff}, \mathit{tt})$ Boolean s.r., $\mathbb{B} = \{\mathit{tt}, \mathit{ff}\}$
- $(\mathrm{N}, +, \cdot, 0, 1)$ natural number s.r.

$(\mathbb{S}, \oplus, \odot, \mathbf{0}, \mathbf{1})$ semiring

Examples of semirings (s.r.):

- $(\mathbb{B}, \vee, \wedge, \mathit{ff}, \mathit{tt})$ Boolean s.r., $\mathbb{B} = \{\mathit{tt}, \mathit{ff}\}$
- $(\mathrm{N}, +, \cdot, 0, 1)$ natural number s.r.
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- every bounded, distributive lattice $(S, \vee, \wedge, 0, 1)$
- every ring, semifield, and field

Example: $\mathcal{P}(\{1,2\}^*)$-wta $\mathcal{A} = (Q, \Sigma, \delta', F)$ with

- $Q = \{q, \overline{\alpha}, f\}$,
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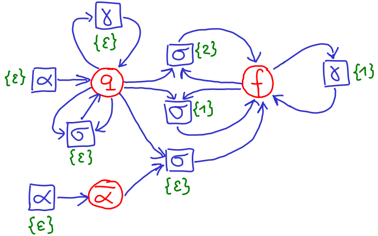

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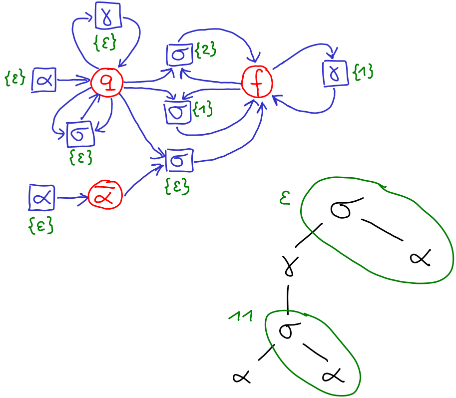

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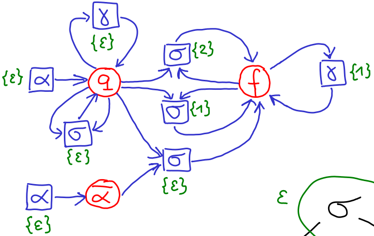

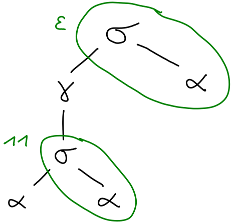

$L_{\mathcal{A}}(\xi) = \{\varepsilon,\ 11\}$
set of reverse positions of $\sigma(., \alpha)$

outline ...

- support of recognizable $\mathbb{S}$-weighted tree languages
- inductive computation of $L_{\mathcal{A}}$
- determinization, minimization, decidability
- characterization results for $\mathrm{REC}(\mathbb{S})$
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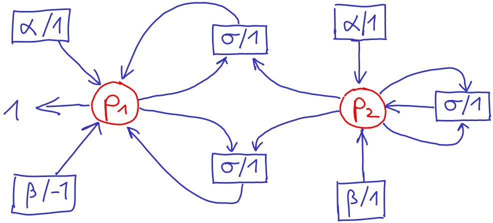
counterexample: [Berstel, Reutenauer 88]

$\mathbb{Z}$-wta $\mathcal{A} = (Q, \Sigma, \delta, F)$



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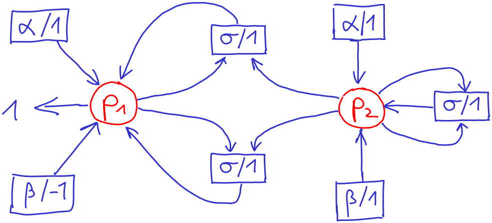

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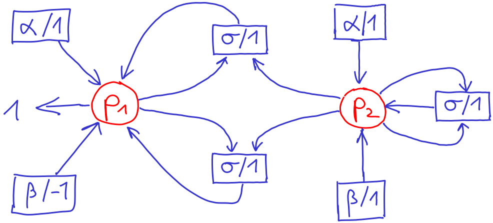

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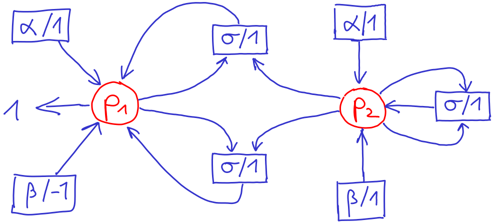

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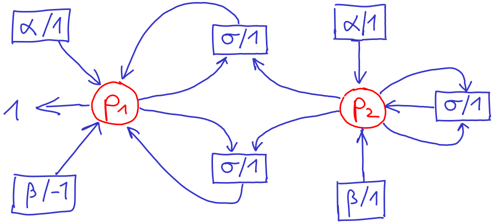

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support of L: $\mathrm{supp}(L) = \{\xi \in T_\Sigma \mid L(\xi) \neq 0\}$

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e.g. not: $\mathbb{Z}$, rings

outline ...

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$\mathbb{S}$-wta $\mathcal{A} = (Q, \Sigma, \delta, F)$, $\quad L_{\mathcal{A}}(\xi) = \bigoplus_{\kappa : \mathrm{pos}(\xi) \to Q} \Big(\mathrm{wt}(\kappa) \odot F(\kappa(\varepsilon)) \Big)$

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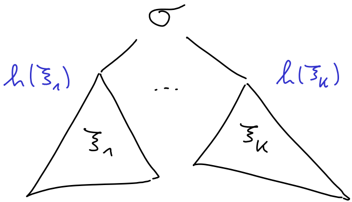

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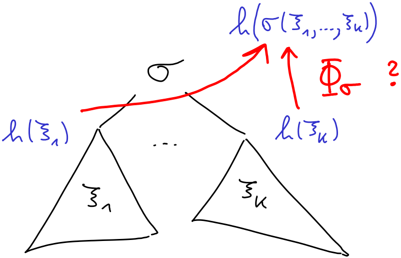

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↑ needs distributivity from right <u>and</u> left

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$\mathcal{A}$ is *deterministic* if

$$\delta(q_1 \ldots q_k, \sigma, q) \neq 0 \text{ and } \delta(q_1 \ldots q_k, \sigma, q') \neq 0 \text{ imply } q = q'.$$

for every ξ there is at most one run κ with $\mathrm{wt}(\kappa) \neq 0$.

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$$\mathcal{A} \text{ is deterministic and } \delta(q_1 \dots q_k, \sigma, q) \in \{0, 1\}.$$

$$L_\mathcal{A} = \bigoplus_{i=1}^{k} s_i \cdot \chi_{L_i} \text{ for some recogn. } L_i \subseteq T_\Sigma \text{ and } s_i \in S.$$

image of $L_\mathcal{A}$ is finite

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image of $L : T_\Sigma \to \mathbb{N}; \xi \mapsto \#_{\sigma(.\alpha)}(\xi)$ is not finite.

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Theorem:	For every ???? s.r. $\mathbb{S}$ and every $\mathbb{S}$-wta there is an equivalent crisp deterministic $\mathbb{S}$-wta.

Theorem: For every locally finite s.r. $\mathbb{S}$ and every $\mathbb{S}$-wta there is an equivalent crisp deterministic $\mathbb{S}$-wta.

$\mathbb{S}$ is locally finite if for every finite $A \subseteq \mathbb{S}$ the set $\langle A \rangle_{+,\cdot,0,1}$ is finite

e.g. every bounded distributive lattice $(L, \vee, \wedge, 0, 1)$ is a locally finite s.r.

Theorem: For every locally finite s.r. $\mathbb{S}$ and every $\mathbb{S}$-wta there is an equivalent crisp deterministic $\mathbb{S}$-wta.

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[Borchardt 04] $\mathbb{S}$ tropical s.r. $(\mathbb{R}_{\geq 0} \cup \{\infty\}, \min, +, \infty, 0)$

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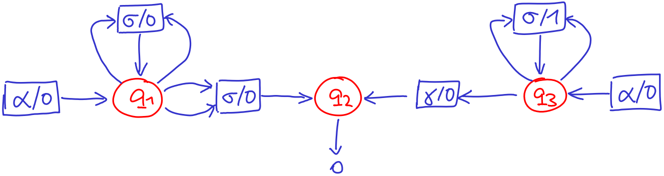

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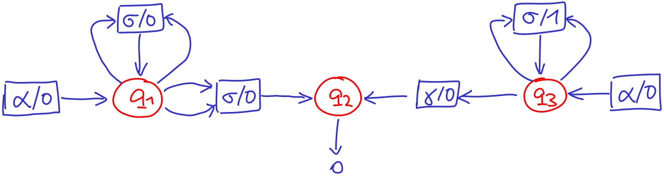

$L : T_\Sigma \to \mathbb{R}_{\geq 0} \cup \{\infty\}$ with

$$L(\xi) = \begin{cases} 0 & \text{if } \xi = \sigma(s,t), \text{ no } \gamma \text{ occurs in } s \text{ and } t \\ \#_\sigma(s) & \text{if } \xi = \gamma(s), \text{ no } \gamma \text{ occurs in } s \\ \infty & \text{otherwise} \end{cases}$$

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Theorem	For every extremal semifield $\mathbb{S}$ and $\mathbb{S}$-wta with the twins property there is an equivalent deterministic $\mathbb{S}$-wta.

[Kirsten, Mäurer 03; Büchse, May, V. 10]

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deterministic wta over semifields:

[Borchardt 03, 04] exponential-time

[Maletti 09] polynomial-time

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decidability of equivalence:

wta over fields: [Seidl 90] polynomial-time

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- weighted regular tree grammar in normal form

e.g. $L_{\#} : T_{\Sigma} \to \mathbb{N}, \quad \xi \mapsto \#_{\sigma(.,\alpha)}$

$$
\begin{aligned}
(q\overline{\alpha}, \sigma, f) &\quad\leadsto\quad f \xrightarrow{1} \sigma(q, \overline{\alpha}) \\
(qq, \sigma, q) &\quad\leadsto\quad q \xrightarrow{1} \sigma(q, q) \\
(qf, \sigma, f) &\quad\leadsto\quad f \xrightarrow{1} \sigma(q, f) \ \ldots
\end{aligned}
$$

$\mathrm{REC}(\mathbb{S})$ is equal to the class of $\mathbb{S}$-weighted tree languages defined by ...

- weighted regular tree grammar in normal form
- weighted regular tree grammar, if $\mathbb{S}$ allows infinite summation [Ésik, Kuich 03]

$q \xrightarrow{s} q$ chain rules

$\mathrm{REC}(\mathbb{S})$ is equal to the class of $\mathbb{S}$-weighted tree languages defined by ...

- weighted regular tree grammar in normal form
- weighted regular tree grammar, if $\mathbb{S}$ allows infinite summation
- weighted rational expressions (Kleene's theorem)
  [Droste, Pech, V. 05; Fülöp, Maletti, V. 09]

e.g. $L_{\#} : T_{\Sigma} \to \mathbb{N}, \quad \xi \mapsto \#_{\sigma(.,\alpha)}$

$$\begin{aligned}\eta &= \eta_1 \circ_z 1.\sigma(z,\alpha) \circ_z \eta_2 \\ \eta_1 &= \big(1.\gamma(z) + 1.\sigma(\eta_2, z) + 1.\sigma(z, \eta_2)\big)^*_z \\ \eta_2 &= \big(1.\gamma(z) + 1.\sigma(z,z)\big)^*_z \circ_z 1.\alpha\end{aligned}$$

$\mathrm{REC}(\mathbb{S})$ is equal to the class of $\mathbb{S}$-weighted tree languages defined by ...

- weighted regular tree grammar in normal form
- weighted regular tree grammar, if $\mathbb{S}$ allows infinite summation
- weighted rational expressions (Kleene's theorem)
- weighted RMSO-formulas (Büchi's theorem) [Droste, V. 06, 11]

e.g. $L_{\#} : T_{\Sigma} \to \mathbb{N}, \quad \xi \mapsto \#_{\sigma(.,\alpha)}$

$\varphi = \exists x.\mathsf{label}_{\sigma}(x) \wedge (\exists y.\mathrm{edge}_2(x,y) \wedge \mathsf{label}_{\alpha}(y))$

$\vee$ and $\exists$ are interpreted by $\oplus$
$\wedge$ and $\forall$ are interpreted by $\odot$ [Droste, Gastin 05]

outline ...

- support of recognizable $\mathbb{S}$-weighted tree languages
- inductive computation of $L_{\mathcal{A}}$
- determinization, minimization, decidability
- characterization results for $\mathrm{REC}(\mathbb{S})$
- weighted tree automata over other algebras

recall: $\kappa : \mathrm{pos}(\xi) \to Q$

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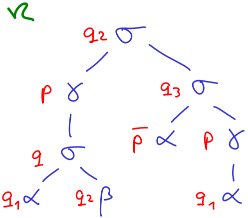
weight of κ:

$$\mathrm{wt}(\kappa) = \bigodot_{w} \delta(\kappa(w1) \dots \kappa(wk), \xi(w), \kappa(w))$$

wta over s.r.



recall: $\kappa : \mathrm{pos}(\xi) \to Q$

weight of κ: $\mathbb{S}$ commutative

$$\mathrm{wt}(\kappa) = \mathrm{wt}(\kappa_1) \odot \ldots \odot \mathrm{wt}(\kappa_k) \quad \odot \quad \delta(\kappa(1)\ldots\kappa(k), \xi(\varepsilon), \kappa(\varepsilon))$$

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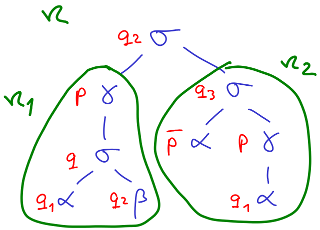

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$$\mathrm{wt}(\kappa) = \underbrace{\delta(\kappa(1) \ldots \kappa(k), \xi(\varepsilon), \kappa(\varepsilon))}_{\text{operation on } \mathbb{S}} \Big(\mathrm{wt}(\kappa_1), \ldots, \mathrm{wt}(\kappa_k)\Big)$$

wta over multioperator monoids $(\mathbb{S}, +, 0, \Omega)$
[Kuich '99, Maletti '04, Fülöp, Maletti, Stüber, V. 09,10]

recall: $\kappa : \mathrm{pos}(\xi) \to Q$

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$\mathrm{wt}(\kappa) = \mathrm{Val}(s_1, \ldots, s_n)$; s_i = weight of transition
wta over valuation monoids $(\mathbb{S}, +, 0, \mathrm{Val})$
[Droste, Meinecke '11]

Theory of $\mathrm{REC}(\mathbb{S})$ depends on the properties of $\mathbb{S}$.

hints for further reading:

- [Gecseg, Steinby '84] Tree Automata, Akademiai Kiado.
- [Gecseg, Steinby '97] Tree Languages, in: Handbook of Formal Languages.
- [Comon et al.] Tree Automata Techniques and Applications (TATA), online.

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- [Golan '99] Semirings and Their Applications.
- [Eilenberg '74] Automata, Languages, and Machines, Vol. A.
- [Sakarovitch '03] Elements of Automata Theory.
- eds. Droste, Kuich, V. '09 Handbook of Weighted Automata.

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- [Golan '99] Semirings and Their Applications.
- [Eilenberg '74] Automata, Languages, and Machines, Vol. A.
- [Sakarovitch '03] Elements of Automata Theory.
- eds. Droste, Kuich, V. '09 Handbook of Weighted Automata.
- [Ésik, Kuich '03] Formal Tree Series.
- [Fülöp, V. '09] Weighted Tree Automata and Tree Transducers.

References:

- [Berstel, Reutenauer 88] Rational Series and Their Languages.
- [Borchardt 03] The Myhill-Nerode theorem for recognizable tree series.
- [Borchardt 04] A pumping lemma and decidability problems for recognizable tree series.
- [Bozapalidis 91] Effective construction of the syntactic algebra of a recognizable series on trees.
- [Bozapalidis, Louskou-Bozapalidou 83] The rank of a formal tree power series.
- [Büchse, May, Vogler 10] Determinization of weighted tree automata using factorizations.
- [Ésik, Kuich 03] Formal Tree Series.
- [Droste, Gastin 05] Weighted automata and weighted logics.
- [Droste, Pech, Vogler 05] A Kleene theorem for weighted tree automata.
- [Droste, Vogler 06] Weighted Tree Automata and Weighted Logics.
- [Droste, Vogler 01] Weighted logics for unranked tree automata.
- [Fülöp, Maletti, Vogler 09] A Kleene theorem for weighted tree automata over distributive multioperator monoids.
- [Kirsten, Mäurer 05] On the determinization of weighted automata.
- [Kuich 99] Linear systems of equations and automata on distributive multioperator monoids.
- [Maletti 04] Relating tree series transducers and weighted tree automata.
- [Maletti 09] Minimizing deterministic weighted tree automata.
- [Seidl 90] Deciding Equivalence of Finite Tree Automata.
- [Stüber, Vogler, Fülöp 09] Decomposition of weighted multioperator tree automata.

TECHNISCHE UNIVERSITÄT DRESDEN

6th International Workshop

Weighted Automata: Theory and Application

May 29 – June 2, 2012, Dresden, Germany

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Important Dates in 2012:
- submission of abstracts: March 16
- notification of acceptance: March 31
- registration: April 27
- workshop: May 29 – June 2
- submission of papers: July 30

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