# Unification in Propositional Logic

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Unification theory plays an essential role in many applications of logic to Computer Science, expecially in Automated Deduction and related areas. $E$-*unification theory* takes care of unification in contexts where syntactic identity is relativized to 'identity modulo' a given equational theory $E$.

Among the relevant theories $E$ for which mgus still exist, we have Boolean algebras. This leads to the following very natural question: what does it happen in case we pass from classical propositional logic (i.e. from Boolean algebras) to non-classical propositional logics (e.g. to Heyting algebras or to modal algebras)? This is the subject of this talk. We shall see that in some relevant cases elementary unification is finitary, although the algorithmic problems become much more intricated. The interesting fact arising is the following: *some important purely logical questions* (such as the problem of recognizing admissible inference rules or the problem of recognizing exact formulas) have a quite natural solution within the context of $E$-unification theory. The area of $E$-unification in propositional logic is nevertheless new, many problems still wait for a solution or for a better solution: these problems include extensions of recent results to unification with constants/general unification and to further natural systems, structural information concerning the relationship between unification types and the the lattice of varieties, improvements of the few existing algorithms from the computational point of view, complexity and implementation questions, etc.

My talk is organized as follows: basic definitions are first recalled and the central role played by finitely presented *projective* algebras is illustrated. Then the case of Heyting algebras is closely investigated (both from a mathematical and from a computational point of view); finally, relevant results are extended to some common modal systems over $K4$.