Practical problems with Chomsky-Schützenberger parsing for weighted multiple context-free grammars[1]

Tobias Denkinger

tobias.denkinger@tu-dresden.de

Institute of Theoretical Computer Science
Faculty of Computer Science
Technische Universität Dresden

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[1]based on T. Denkinger (2017). "Chomsky-Schützenberger parsing for weighted multiple context-free languages".

The problem: k-best parsing

parsing problem

Input:

- a grammar G
- a word w

Output:

- a derivation of w in G (not unique)

The problem: k-best parsing

k-best parsing problem [Huang and Chiang 2005]

Input:

- a weighted grammar G
- a suitable partial order $\trianglelefteq$ on the weights
- a number $k \in \mathbb{N}$
- a word w

Output:

- a sequence of k best derivations[2] of w in G

(not unique)

[2] w.r.t. weights and $\trianglelefteq$ (less is better)

Multiple context-free grammars

context-free grammars

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multiple context-free grammars [Seki, Matsumura, Fujii, and Kasami 1991]

$$A \to [\underbrace{((x_1, x_2), (y_1, y_2)) \mapsto (\mathrm{a}x_1y_2\mathrm{b}, y_1\mathrm{c}x_2)}_{(\Sigma^* \times \Sigma^*) \times (\Sigma^* \times \Sigma^*) \to (\Sigma^* \times \Sigma^*)}](A, B)$$

composes *tuples* of strings

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$\Longrightarrow$ extra expressive power useful for natural language processing

The Chomsky-Schützenberger theorem

CS-theorem$_s$ [Chomsky and Schützenberger 1963]

Let L be a language. T.f.a.e.

1. $\exists$ CFG G s.t. $L = \mathrm{L}(G)$
2. $\exists$ regular language R,
   $\exists$ Dyck language D,
   $\exists$ homomorphism h
   s.t. $L = h(R \cap D)$

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CS-theorems [Chomsky and Schützenberger 1963] [Yoshinaka, Kaji, and Seki 2010]

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Idea [Hulden 2011, for CFGs]

Use the decomposition provided by (1. $\rightarrow$ 2.) for parsing.

From the CS-theorem to CS-parsing

$$w \in \mathrm{L}(G)$$

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Observation

Each $u \in R \cap D$ encodes a derivation of G.

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$$\mathrm{parse}_{G,\mathit{wt},k}(w)$$

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Derivations and bracket words

$$\alpha: S \to [\ x_1 \quad x_2\ ](A)$$

$$\beta: A \to [\ \mathrm{a}\ \ x_1\ \ b\ \ ,\ \ c\ \ x_2\ ](A)$$

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$$\gamma: A \to [\ \varepsilon\ \ ,\ \ \varepsilon\ ]()$$

word $w =$

Derivations and bracket words

automaton for R:

$$\overset{S_1\bullet}{} \qquad \overset{\bullet\bar{S}_1}{}$$

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$$A_1\bullet \qquad \bar{A}_1\bullet \ \bullet A_2 \qquad \bullet\bar{A}_2$$

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Derivations and bracket words

$S_1 \bullet \qquad\qquad \bullet \bar{S_1}$

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$A_1 \bullet \qquad \bar{A_1}\bullet \;\; \bullet A_2 \qquad \bullet \bar{A_2}$

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Derivations and bracket words

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S_1 ... $\bar{S}_1$

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A_1 ... $\bar{A}_1$ A_2 ... $\bar{A}_2$

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$A_1 \bullet \bar{A}_1 \bullet \quad \bullet A_2 \bullet \bar{A}_2$

word $w =$

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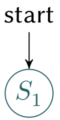

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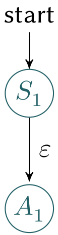

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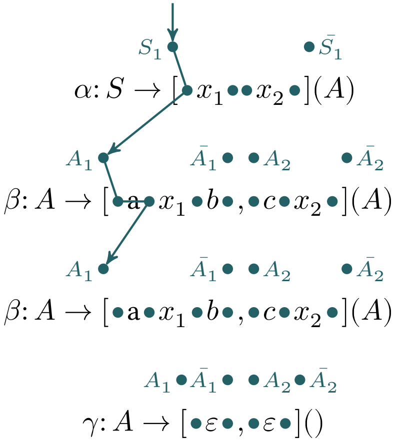

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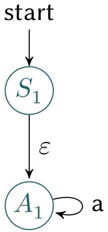
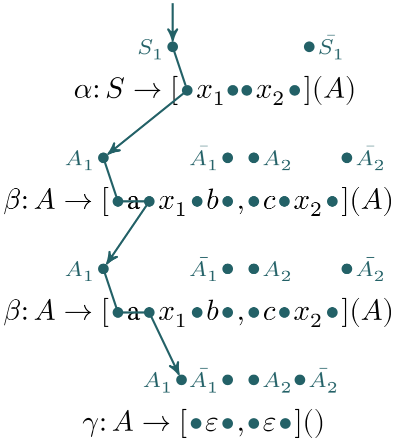
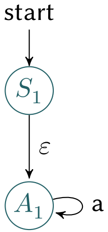


word $w = \mathrm{a}$

Derivations and bracket words

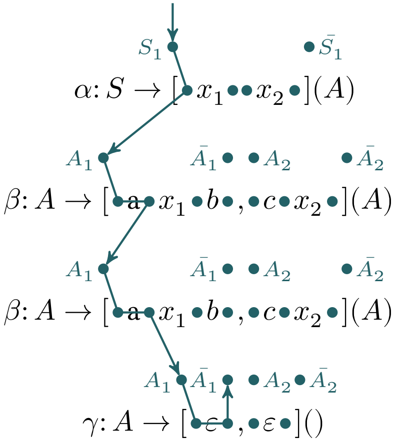


automaton for R:



word $w = \mathrm{aa}$

Derivations and bracket words



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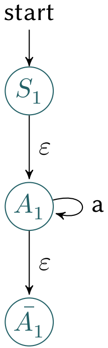

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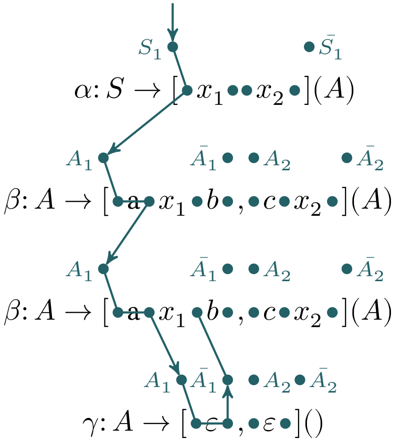

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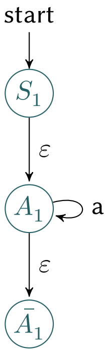

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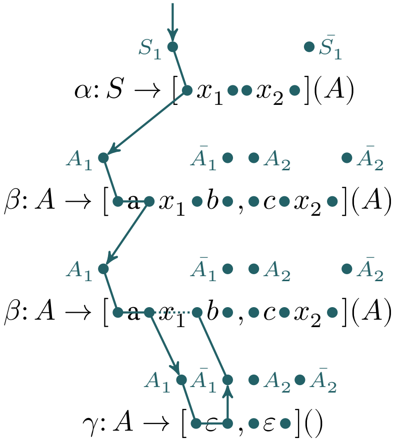

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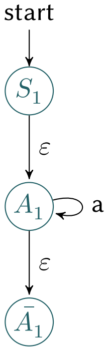
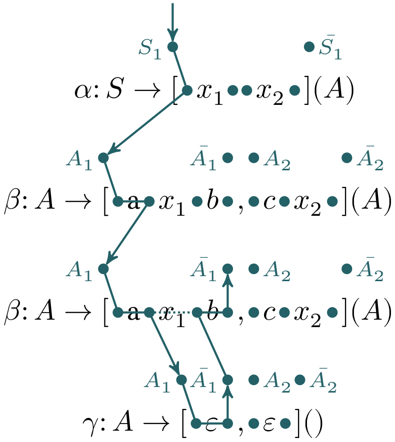
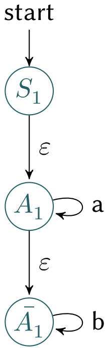


word $w = \text{aa}$

Derivations and bracket words

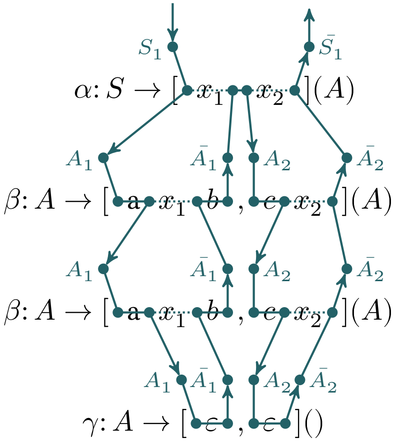


automaton for R:

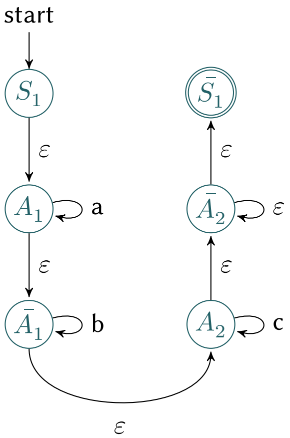


word $w = \text{aab}$

Derivations and bracket words

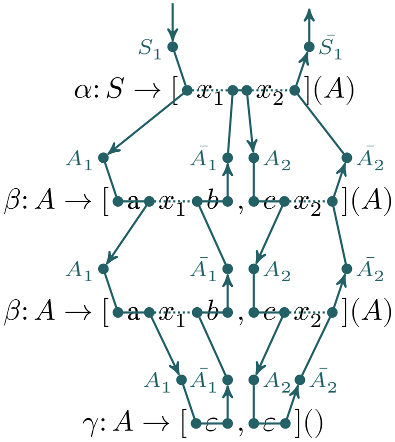


automaton for R:



word $w = \text{aabbcc}$

Derivations and bracket words



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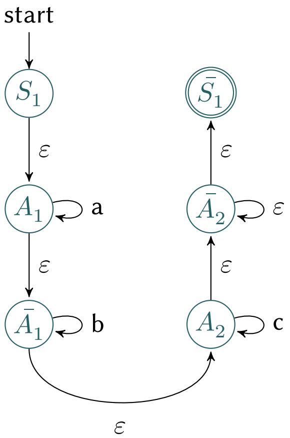
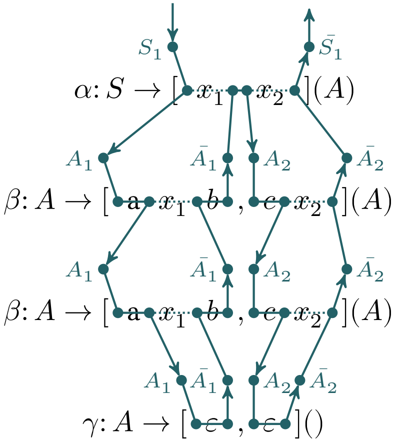


word $w = \text{aabbcc}$

$w' = \text{aaabbcc} \notin \mathrm{L}(G)$

Derivations and bracket words



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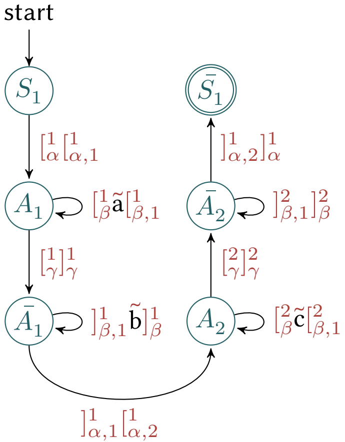

word $w = \mathrm{aabbcc}$ $\qquad$ $w' = \mathrm{aaabbcc} \notin \mathrm{L}(G)$

word $u = [^1_\alpha [^1_{\alpha,1}\ [^1_\beta \tilde{\mathrm{a}} [^1_{\beta,1}\ [^1_\beta \tilde{\mathrm{a}} [^1_{\beta,1}\ [^1_\gamma]^1_\gamma\ ]^1_{\beta,1} \tilde{\mathrm{b}}]^1_\beta\ ]^1_{\beta,1} \tilde{\mathrm{b}}]^1_\beta\ ]^1_{\alpha,1}$
$[^1_{\alpha,2}\ [^2_\beta \tilde{\mathrm{c}} [^2_{\beta,1}\ [^2_\beta \tilde{\mathrm{c}} [^2_{\beta,1}\ [^2_\gamma]^2_\gamma\ ]^2_{\beta,1}]^2_\beta\ ]^2_{\beta,1}]^2_\beta\ ]^1_{\alpha,2}]^1_\alpha$

Practical problems

... with the implementation of $\text{sort}^{wt}_{\trianglelefteq}(R \cap h^{-1}(w))$

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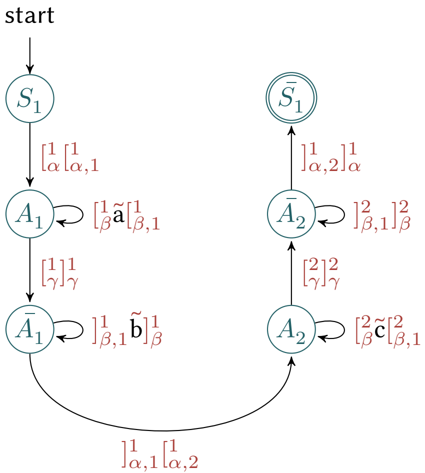

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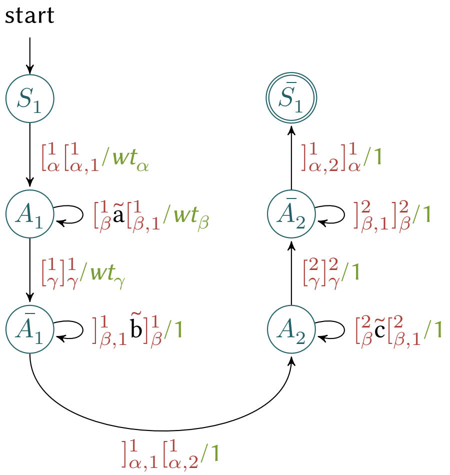
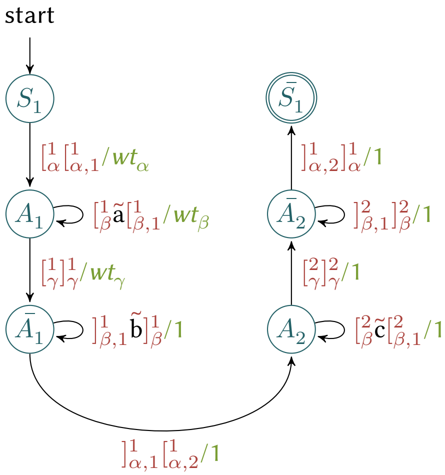


initial idea:

- attach weights to $[^1_\sigma$-brackets

Practical problems

... with the implementation of $\mathrm{sort}^{wt}_{\trianglelefteq}(R \cap h^{-1}(w))$



initial idea:

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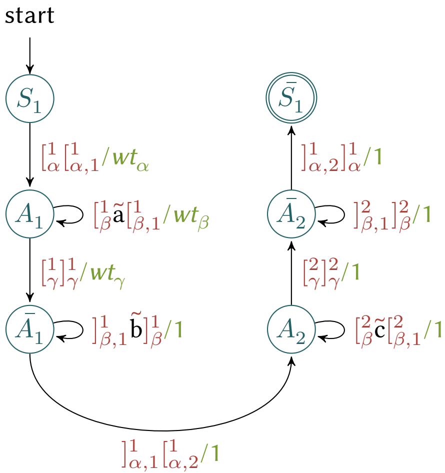
problem:

- loops with weight 1

Solutions and workarounds I



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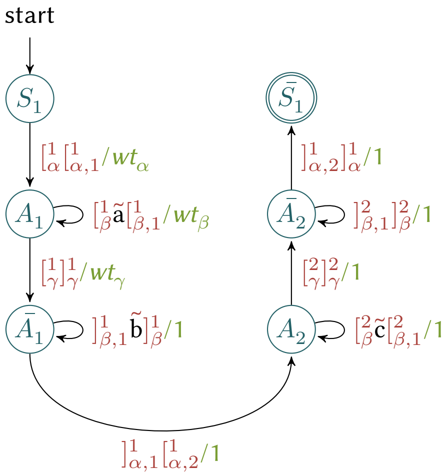
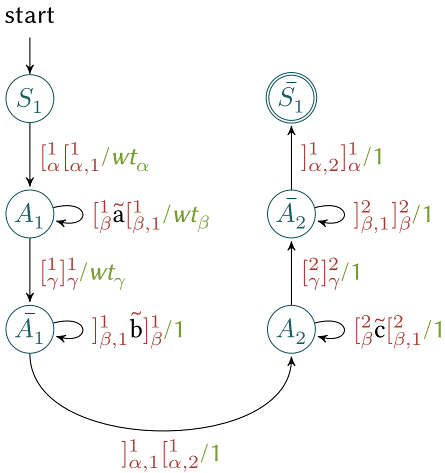


- **assume** that $wt_\sigma \neq 1$ in loops

Solutions and workarounds I



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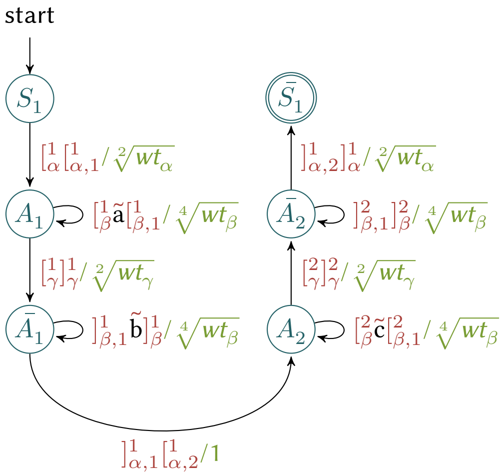

- **assume** that $wt_\sigma \neq 1$ in loops
- **assume** that weights can be *factorised*
- distribute factors of wt_σ among transitions with $[^1_\sigma,]^1_\sigma, [^2_\sigma,]^2_\sigma, \ldots$

Solutions and workarounds II

Assumption 1: weights wt_σ that occur in loops are $\neq 1$

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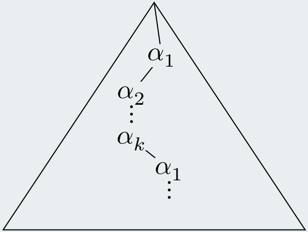
Solutions and workarounds II

Assumption 1: weights wt_σ that occur in loops are $\neq 1$
$\Longrightarrow$ restrict the grammar

restricted weighted MCFGs:

may not have derivations of the form



with $wt_{\alpha_1} = \ldots = wt_{\alpha_k} = 1$

Solutions and workarounds II

Assumption 1: weights wt_σ that occur in loops are $\neq 1$
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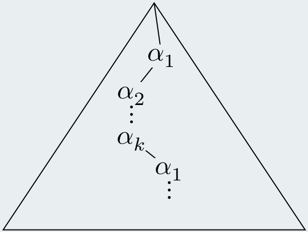

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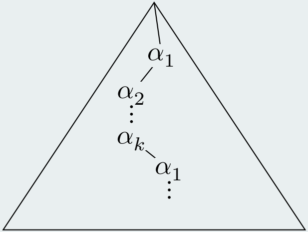

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example algebra	**factorisation**
$(\mathbb{R}_{\geq 0}, \cdot, 1, 0)$	$a = \sqrt[2]{a} \cdot \sqrt[2]{a}$
$(\mathbb{R}_{\geq 0}^{\infty}, \cdot, 1, \infty)$	$a = \sqrt[2]{a} \cdot \sqrt[2]{a}$
$(\mathbb{R}_{\geq 0}^{-\infty}, +, 1, -\infty)$	$a = a/2 + a/2$

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Thank you for your attention.

References

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