

# Coarse-to-fine recognition for weighted tree-stack automata

Max Korn

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## Motivation

- ▶ Problem: Parsing with complicated grammars and recognition with complicated automata are time intensive

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## Motivation

- ▶ Problem: Parsing with complicated grammars and recognition with complicated automata are time intensive
- ▶ Example: multiple-context-free grammars and tree-stack automata with restricted fanout  $k$  have a complexity of  $\mathcal{O}(n^{3k})$
- ▶ Solution: use less complex grammar/ automaton

# Outline

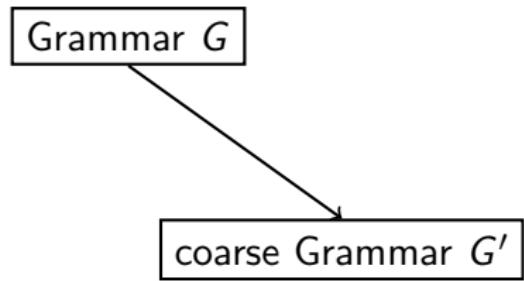
# Outline

# Grammar Based Coarse-to-Fine Parsing

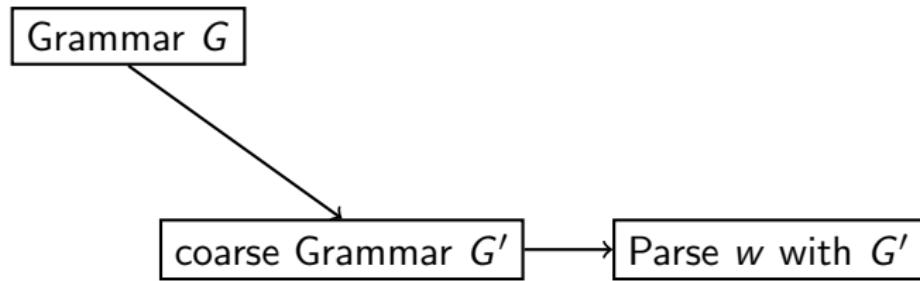
# Grammar Based Coarse-to-Fine Parsing

Grammar  $G$

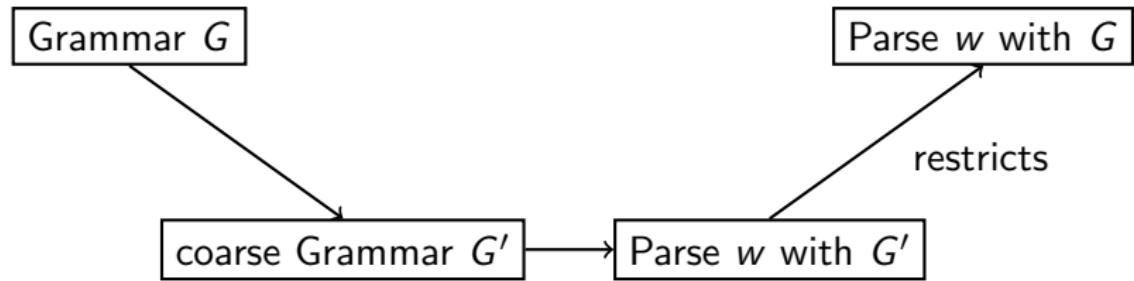
# Grammar Based Coarse-to-Fine Parsing



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## Grammar Based Coarse-to-Fine Parsing



# Data-Storage

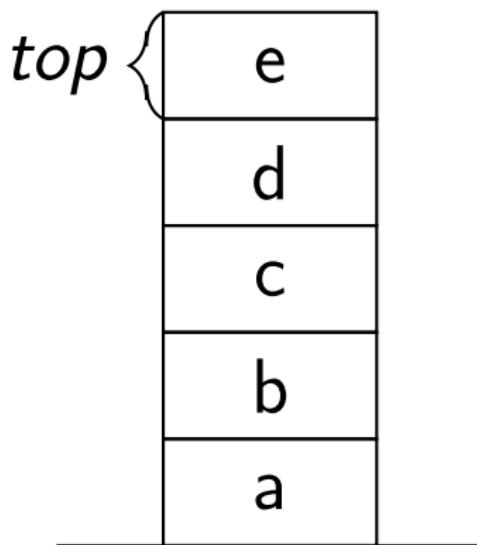
A tuple  $S = (C, P, R, c_i)$  with

# Data-Storage

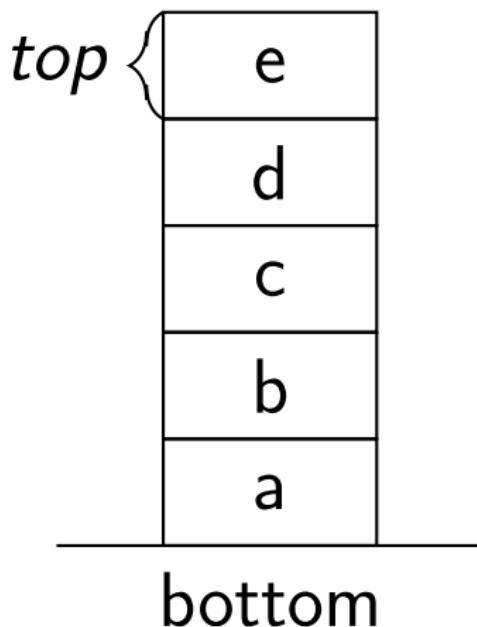
A tuple  $S = (C, P, R, c_i)$  with

- ▶ set  $C$  (of configurations)
- ▶ set  $P$  (of predicates) with  $P \subseteq P(C)$
- ▶ set  $R$  (of instructions) with  $R \subseteq P(C \times C)$
- ▶ initial configuration  $c_i \in C$

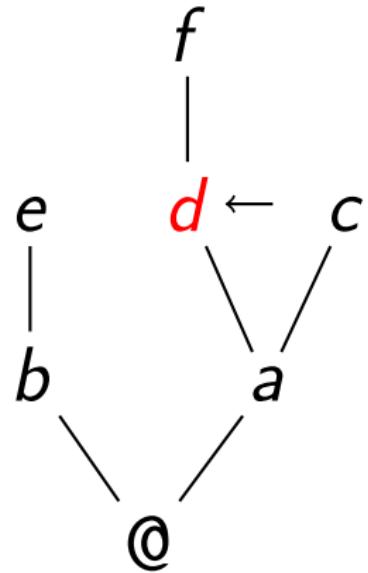
## Push-Down



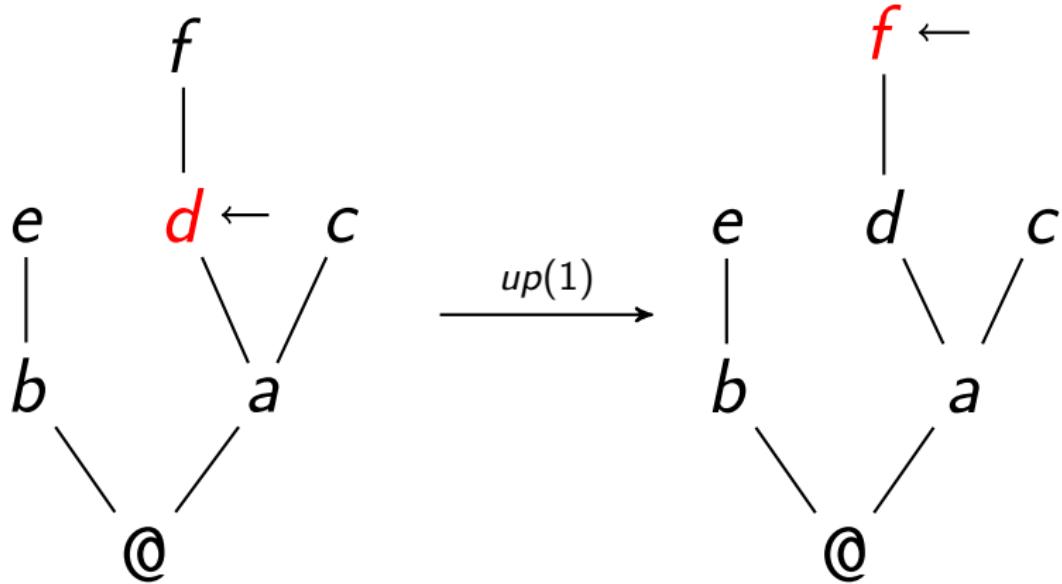
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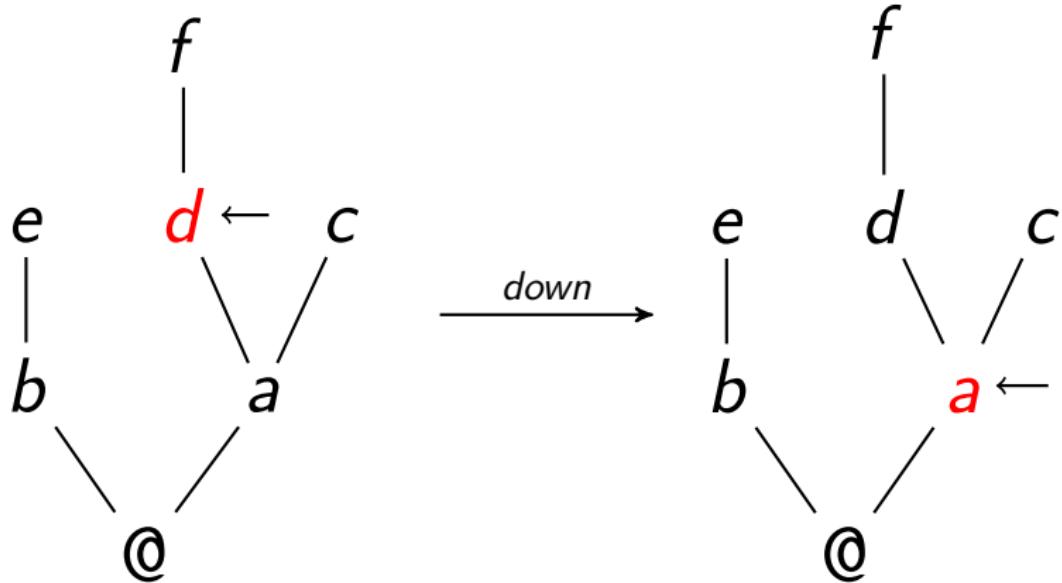
## Tree-Stack



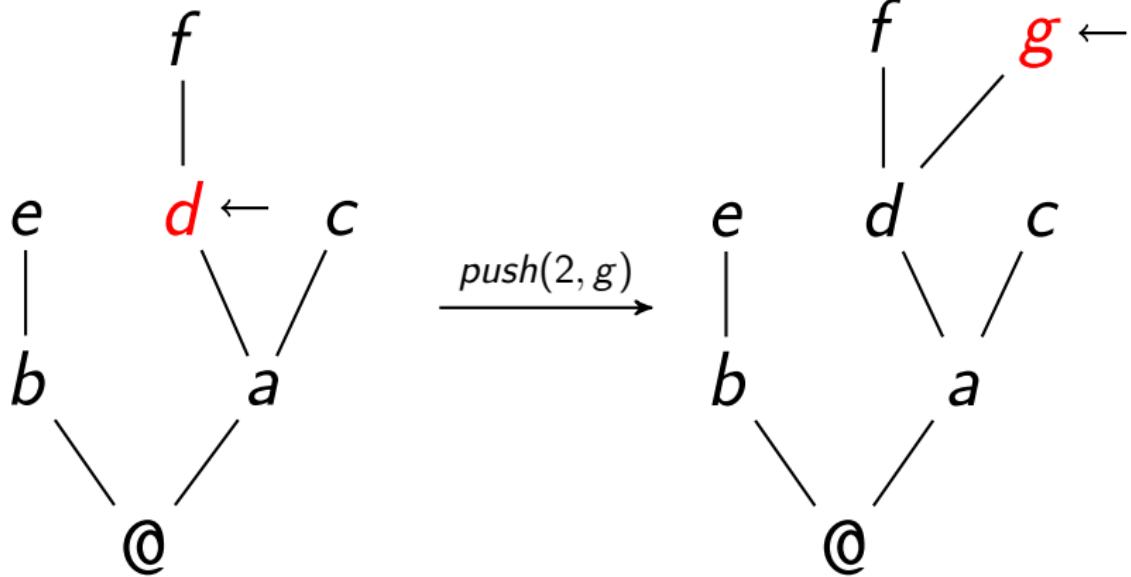
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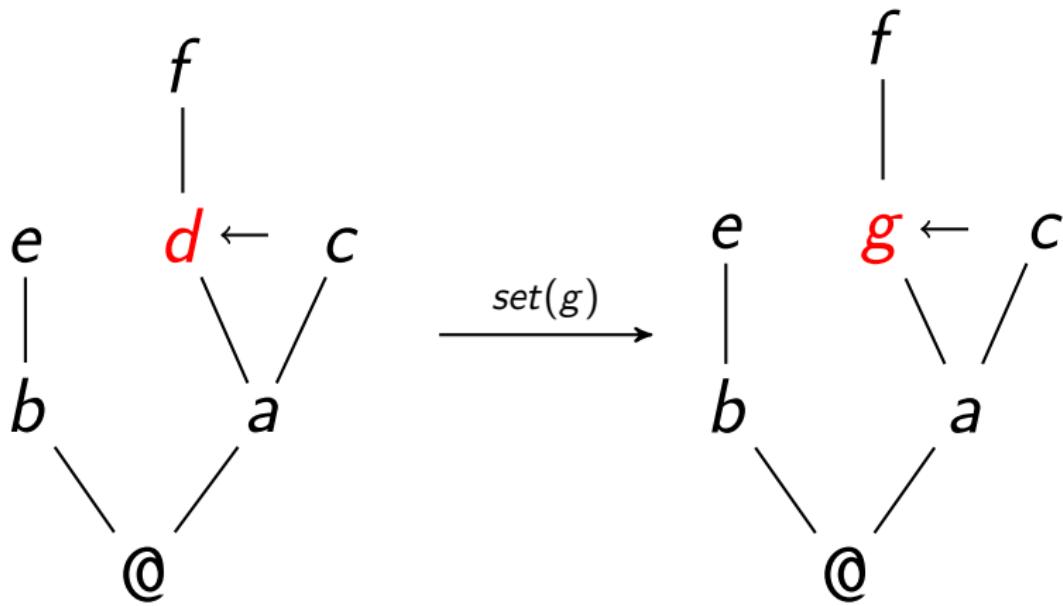
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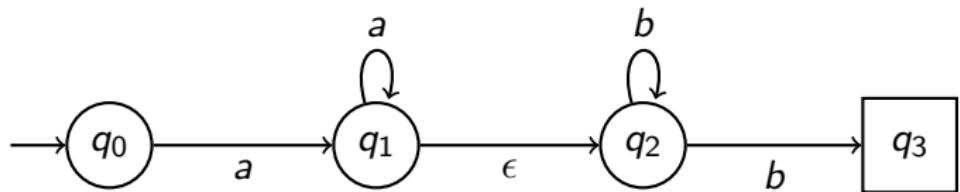
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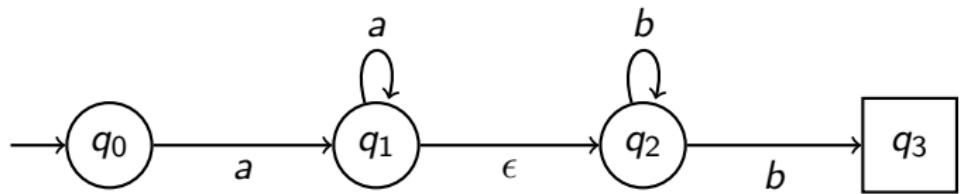
## Tree-Stack



# Automata

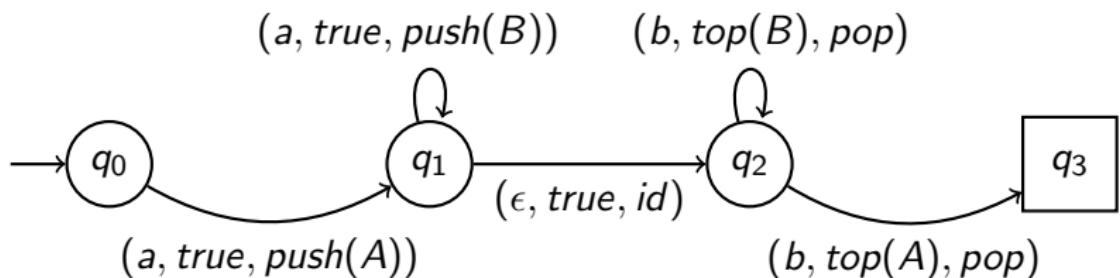


# Automata

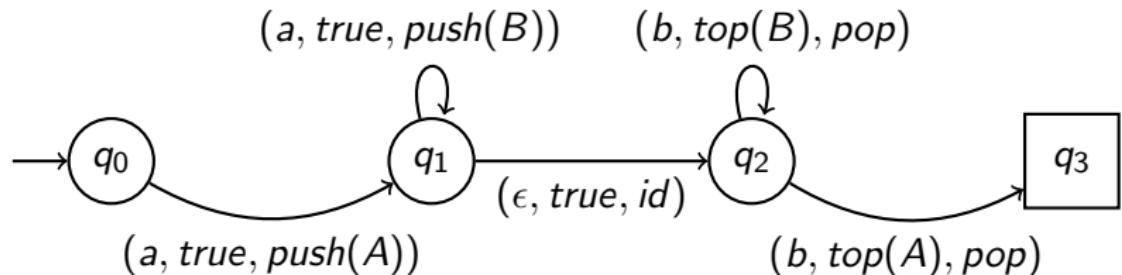


$$L(\mathcal{M}) = \{a^n b^m \mid n, m \geq 1\}$$

# Automata with Storage

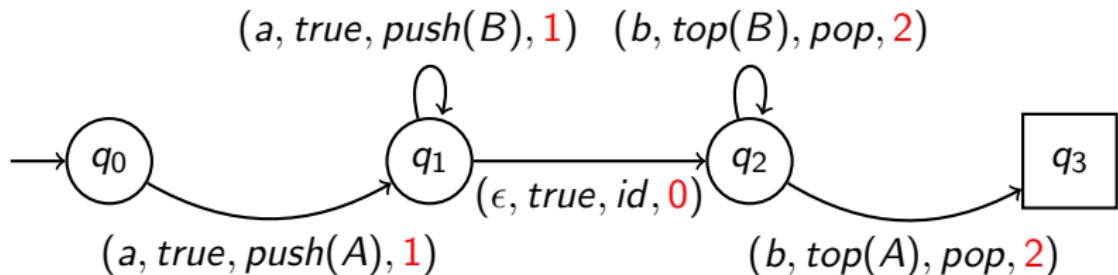


# Automata with Storage



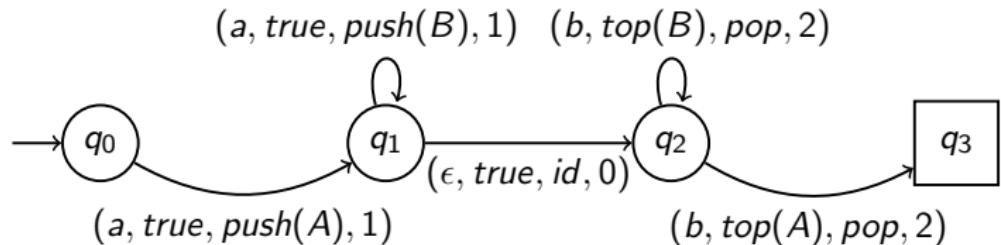
$$L(\mathcal{M}_S) = \{a^n b^n \mid n \geq 1\}$$

# Automata with Storage

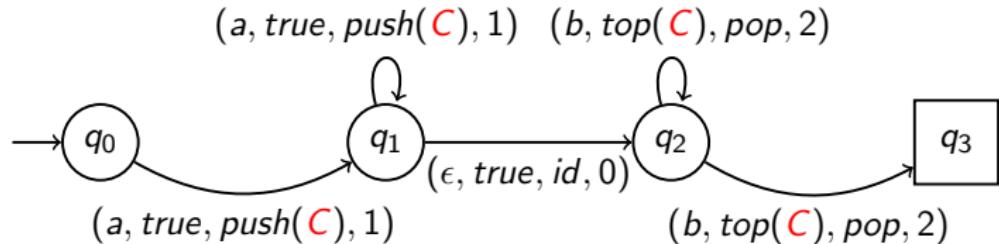
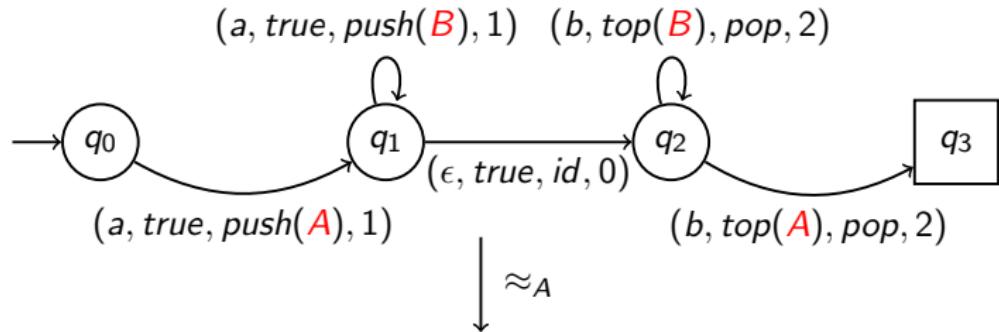


$$L(\mathcal{M}_S) = \{a^n b^n \mid n \geq 1\}$$

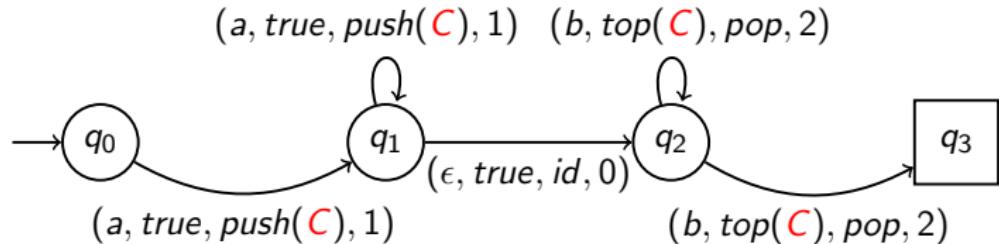
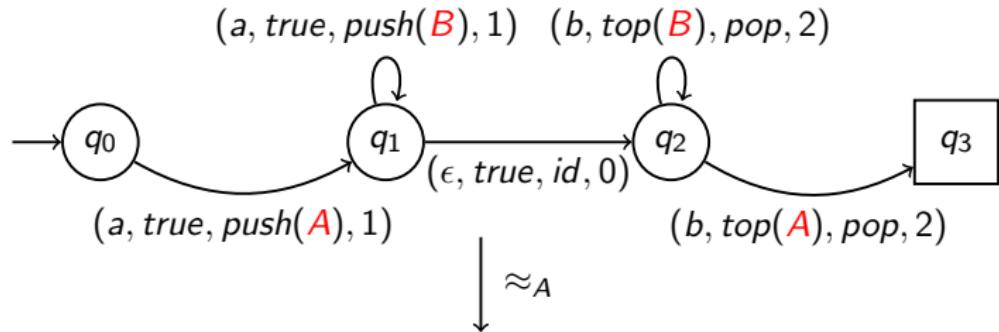
# Approximation



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$$L(\mathcal{M}_{\approx_A}) = \{a^n b^m \mid n \geq m \geq 1\}$$

# Outline

We use three strategies to approximate the initial automaton:

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- ▶ Ignoring tree-structures inspired by Burden and Ljunglöf [1] and Cranenburgh [3] (*TTS*)

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- ▶ Relabelling to equivalence classes of stack symbols by Charniak et al. [2] (*RLB*)

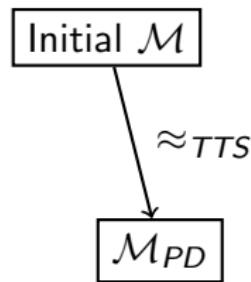
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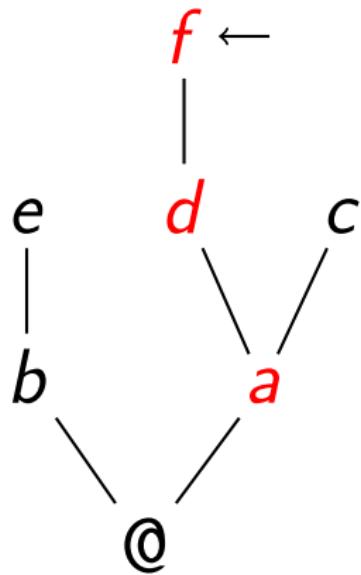
- ▶ Ignoring tree-structures inspired by Burden and Ljunglöf [1] and Cranenburgh [3] (*TTS*)
- ▶ Relabelling to equivalence classes of stack symbols by Charniak et al. [2] (*RLB*)
- ▶ Reducing the amount of push-down configurations to a finite number by Nederhof [5] (*PTK*)

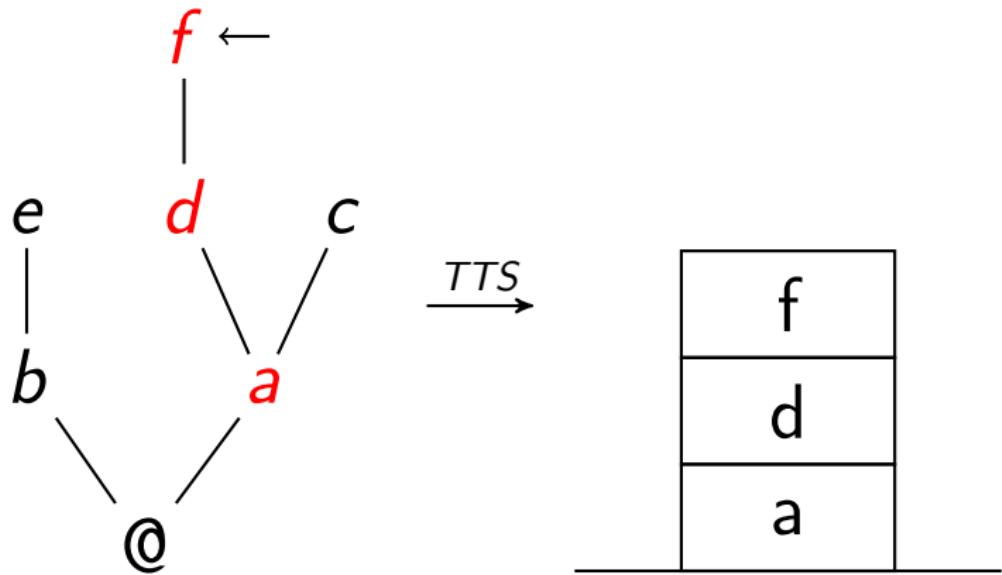
# Ignore Tree-Structure

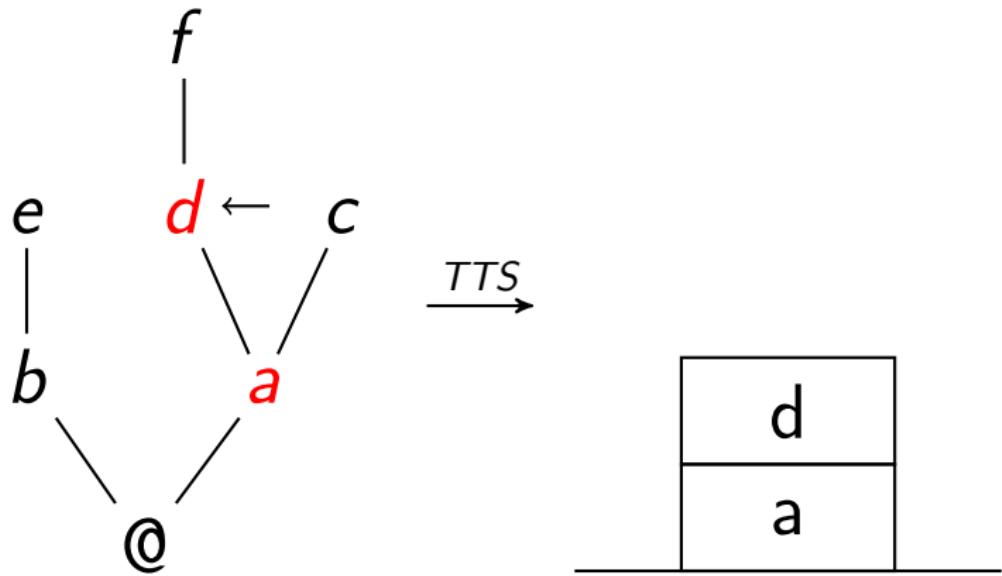
Initial  $\mathcal{M}$

## Ignore Tree-Structure

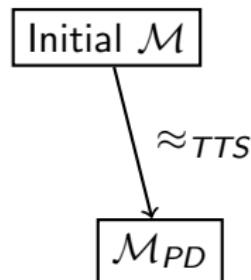




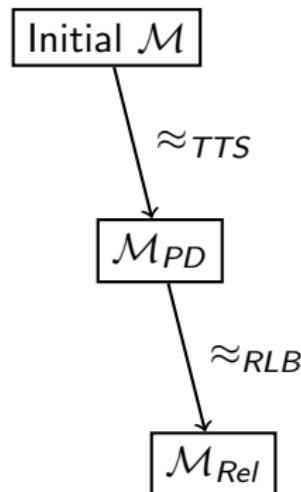


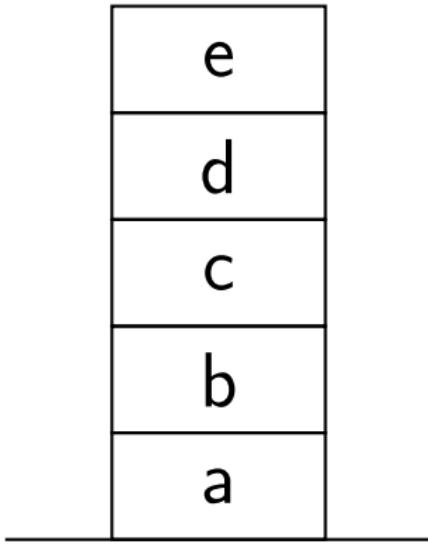


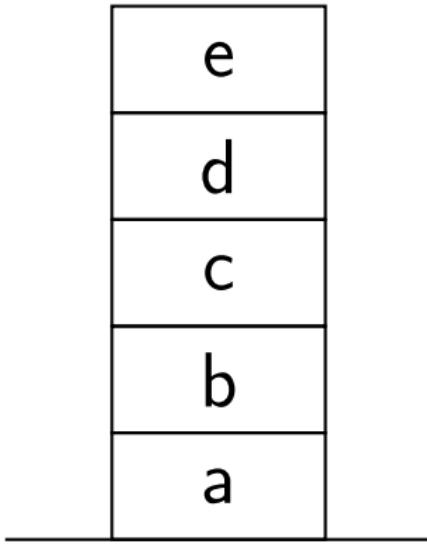
## Equivalence-Classes of Stack symbols



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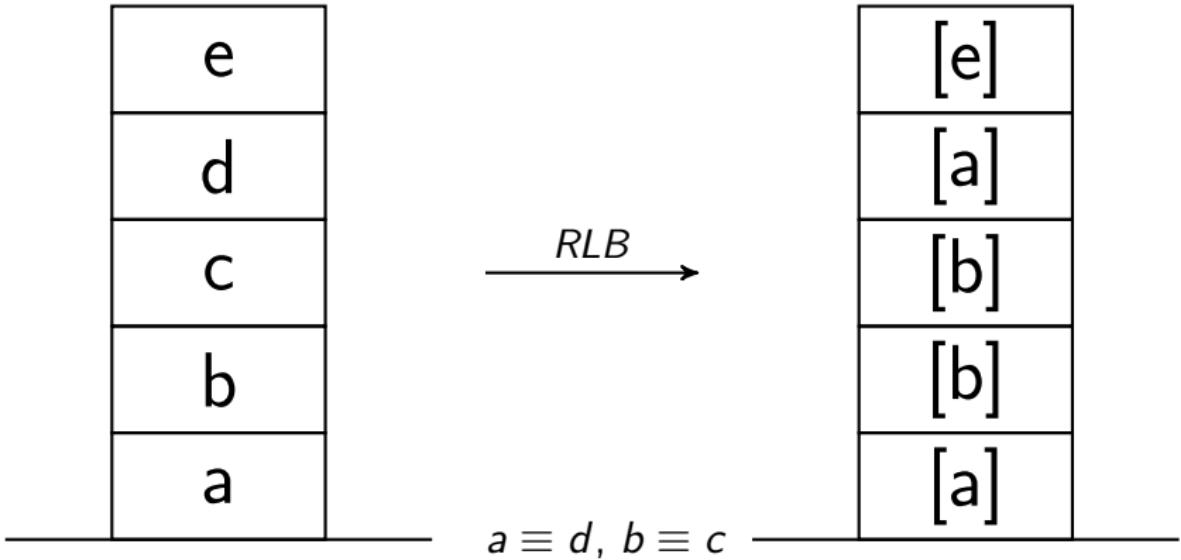




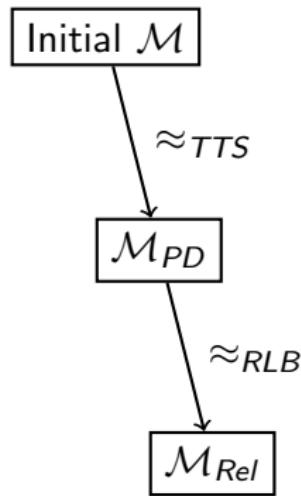


$\xrightarrow{RLB}$

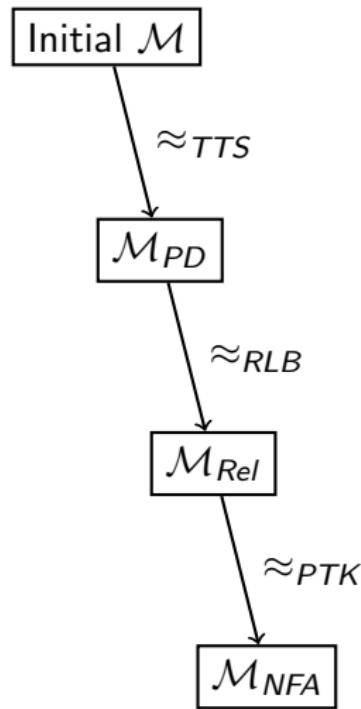
$a \equiv d, b \equiv c$

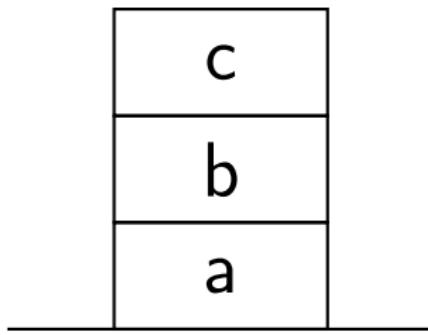


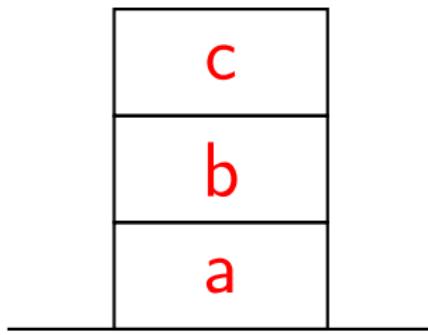
# Limit Push-Down Height

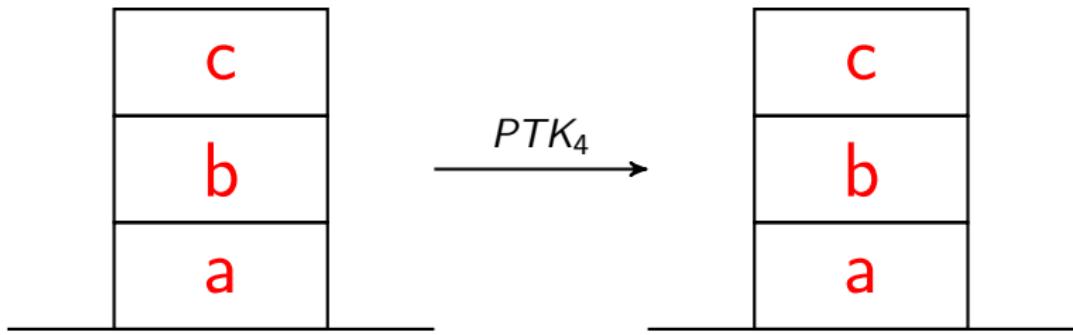


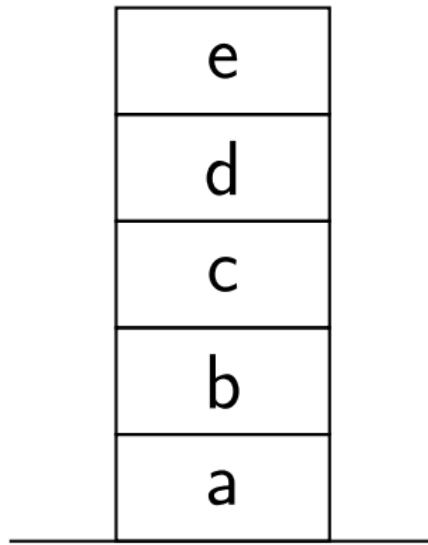
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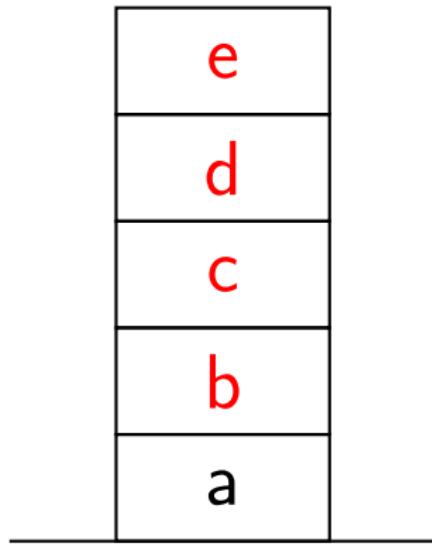


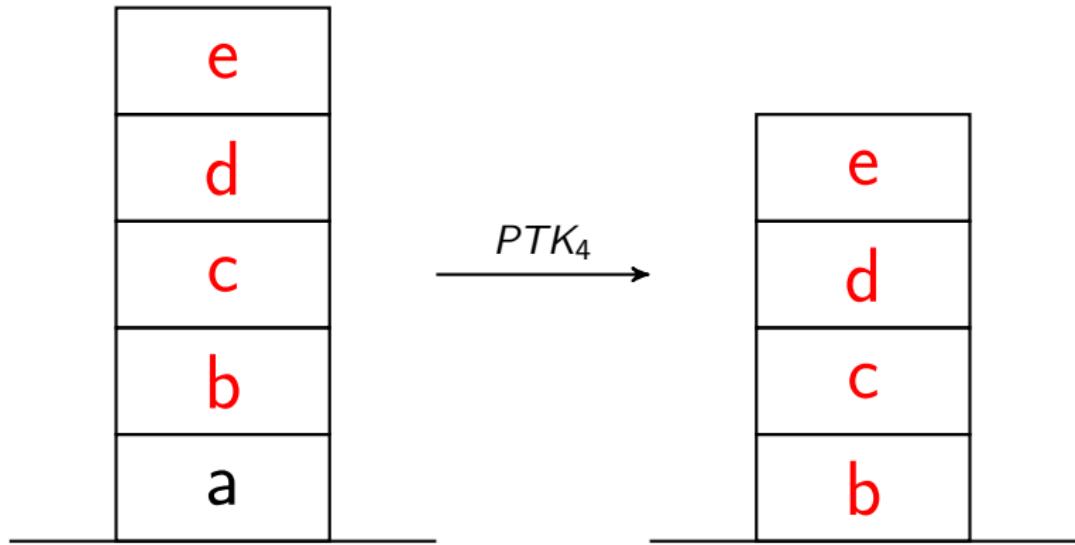






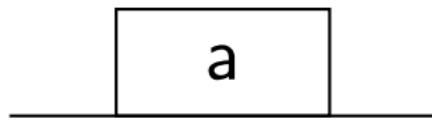






## Special Case

pushdown



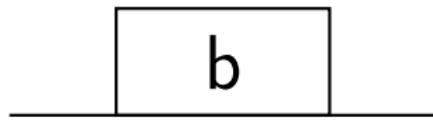
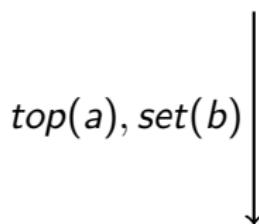
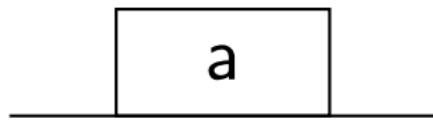
$PTK_4(\text{pushdown})$



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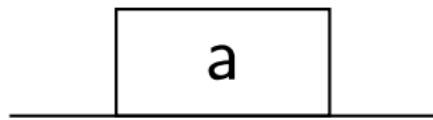
pushdown

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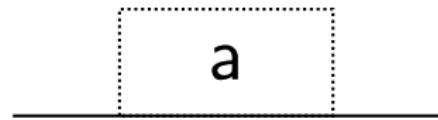


## Special Case

pushdown



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$\text{top}(a), \text{set}(b)$



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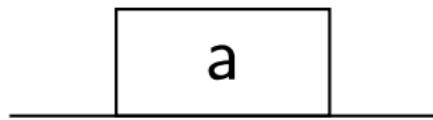


b

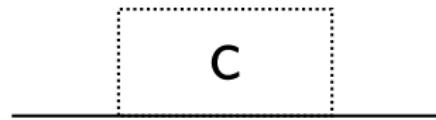
b

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pushdown



$PTK_4(\text{pushdown})$



$\text{top}(a), \text{set}(b)$



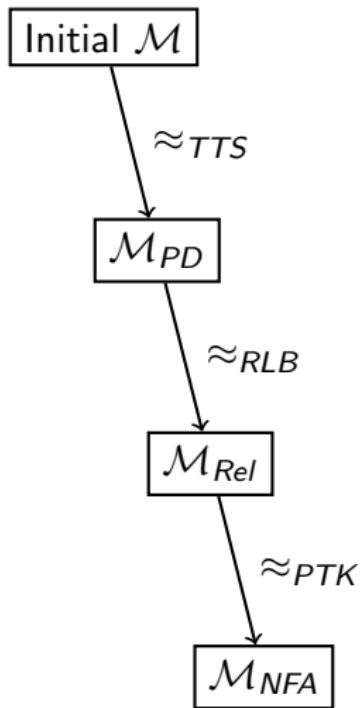
b

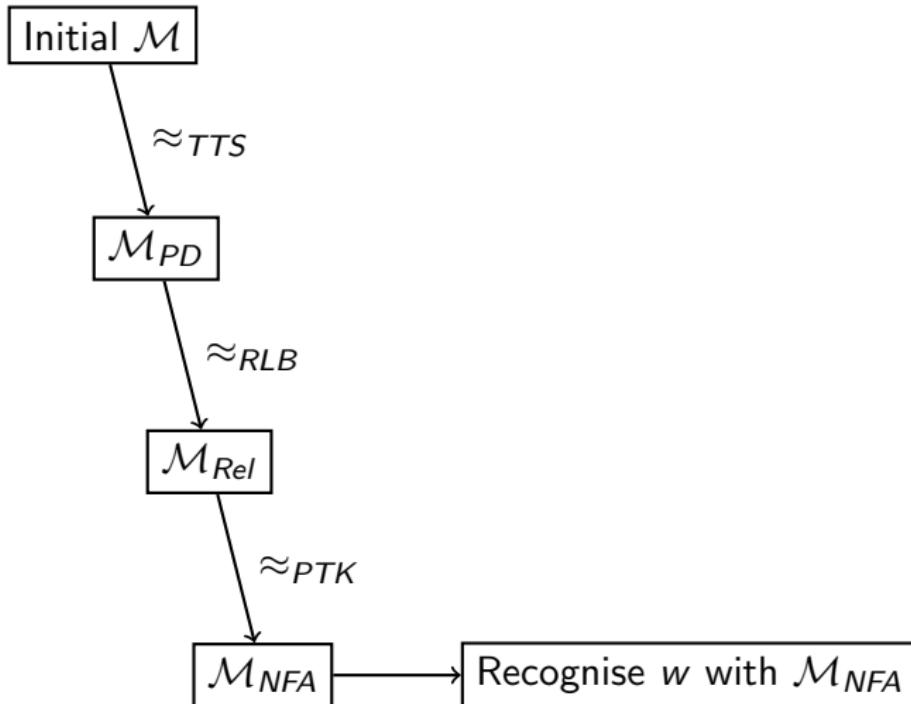
$PTK_4(\text{top}(c), \text{set}(d))$

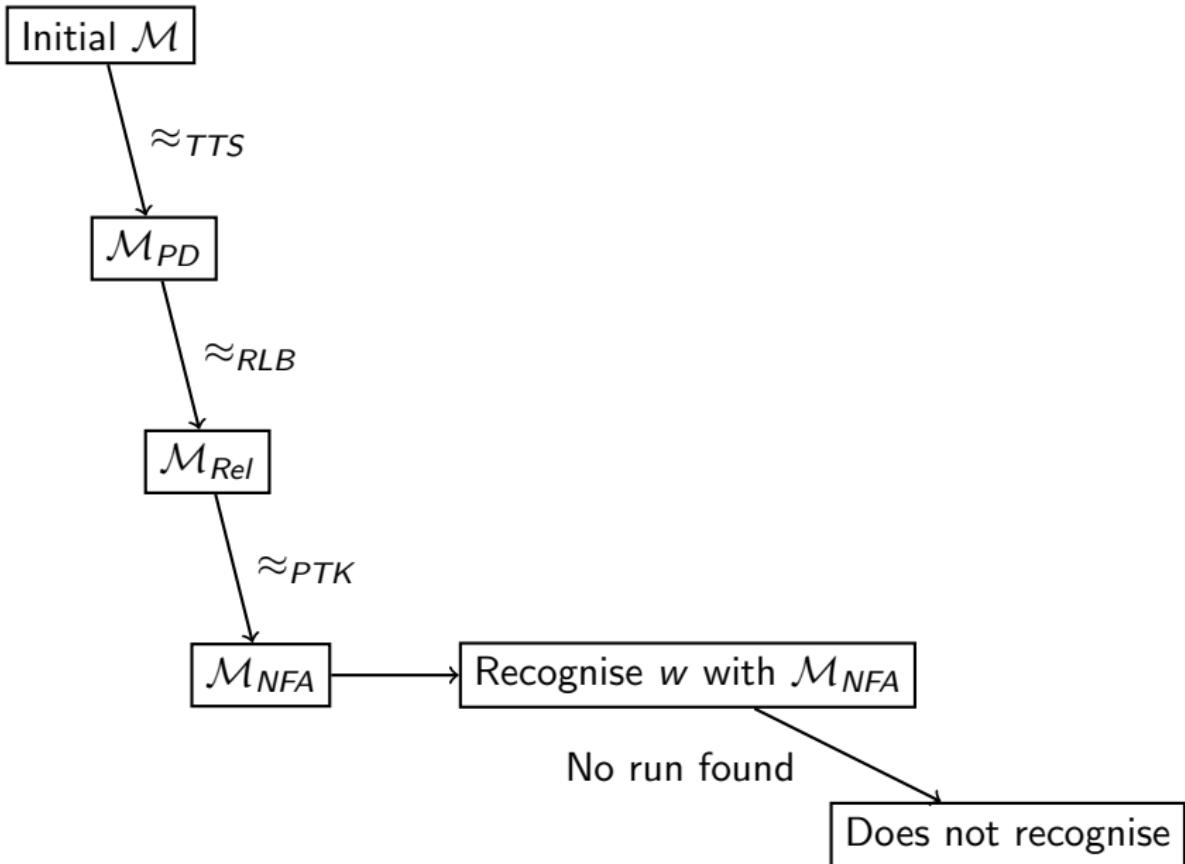


d

# Outline







## Algorithm by Denkinger [4]

**Input:** proper total approximation strategy  $A$ ,

$(S, \Sigma, K)$ -Automaton  $\mathcal{M}$ ,  $n \in \mathbb{N}$ , word  $w \in \Sigma^*$  and part. order  
 $(\leq) \subseteq K \times K$

**Output:** some set of  $n$ -best runs of  $\mathcal{M}$  on  $w$

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1:  $\mathcal{M}' \leftarrow \approx_A(\mathcal{M})$

2:  $P_f \leftarrow \emptyset$

3:  $P_c \leftarrow R_{\mathcal{M}'}(w)$

9: **return**  $P_f$

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4: **while**

**do**

5:      $\theta \leftarrow$  smallest element of  $P_c$

6:      $P_c \leftarrow P_c \setminus \{\theta\}$

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3:  $P_c \leftarrow R_{\mathcal{M}'}(w)$ 
4: while do
5:    $\theta \leftarrow$  smallest element of  $P_c$ 
6:    $P_c \leftarrow P_c \setminus \{\theta\}$ 
7:   for  $\theta' \in \approx_A^{-1}(\theta)$  do
8:     if  $\theta' \in R_{\mathcal{M}}$  then  $P_f \leftarrow P_f \cup \{\theta'\}$ 
9: return  $P_f$ 
```

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- 4: **while**  $|P_f| < n$  or  $\max_{\theta \in P_f} wt(\theta) > \min_{\theta' \in P_c} wt(\approx_A^{-1}(\theta'))$  **do**
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## Algorithm for multiple layers

$$\mathcal{M} \xrightarrow{\approx_{A_1}} \mathcal{M}_1 \xrightarrow{\approx_{A_2}} \mathcal{M}_2 \xrightarrow{\approx_{A_3}} \dots \xrightarrow{\approx_{A_m}} \mathcal{M}_m$$

## Algorithm for multiple layers

$$\mathcal{M} \xrightarrow{\approx_{A_1}} \mathcal{M}_1 \xrightarrow{\approx_{A_2}} \mathcal{M}_2 \xrightarrow{\approx_{A_3}} \dots \xrightarrow{\approx_{A_m}} \mathcal{M}_m$$

**Input:**  $(S, \Sigma, K)$ -Automaton  $\mathcal{M}$ ,  $n \in \mathbb{N}$ , word  $w \in \Sigma^*$ , part. order  $(\leq) \subseteq K \times K$ ,

proper total approximation strategy  $A_1$ ,

proper total approximation strategy  $A_2$ ,

proper total approximation strategy  $A_3$ ,

$\dots$ ,

proper total approximation strategy  $A_m$ ,

**Output:** some set of  $n$ -best runs of  $\mathcal{M}$  on  $w$

## Algorithm for multiple layers

- 1:  $\mathcal{M}_1 \leftarrow \approx_{A_1} (\mathcal{M})$
  - 2:  $\mathcal{M}_2 \leftarrow \approx_{A_2} (\mathcal{M}_1)$
  - ...
  - 3:  $\mathcal{M}_m \leftarrow \approx_{A_m} (\mathcal{M}_{m-1})$
  - 4:  $P_f \leftarrow \emptyset$
  - 5:  $P_m \leftarrow R_{\mathcal{M}_m}(w)$
- 15: **return**  $P_f$

## Algorithm for multiple layers

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- 2:  $\mathcal{M}_2 \leftarrow \approx_{A_2}(\mathcal{M}_1)$
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- 4:  $P_f \leftarrow \emptyset$
- 5:  $P_m \leftarrow R_{\mathcal{M}_m}(w)$
- 6: **while**  $|P_f| < n$  or  $\max_{\theta \in P_f} wt(\theta) > \min_{\theta' \in P_m} wt(\approx_A^{-1}(\theta'))$  **do**
- 7:      $\theta_m \leftarrow$  smallest element of  $P_m$
- 8:      $P_m \leftarrow P_m \setminus \{\theta_m\}$
  
- 15: **return**  $P_f$

## Algorithm for multiple layers

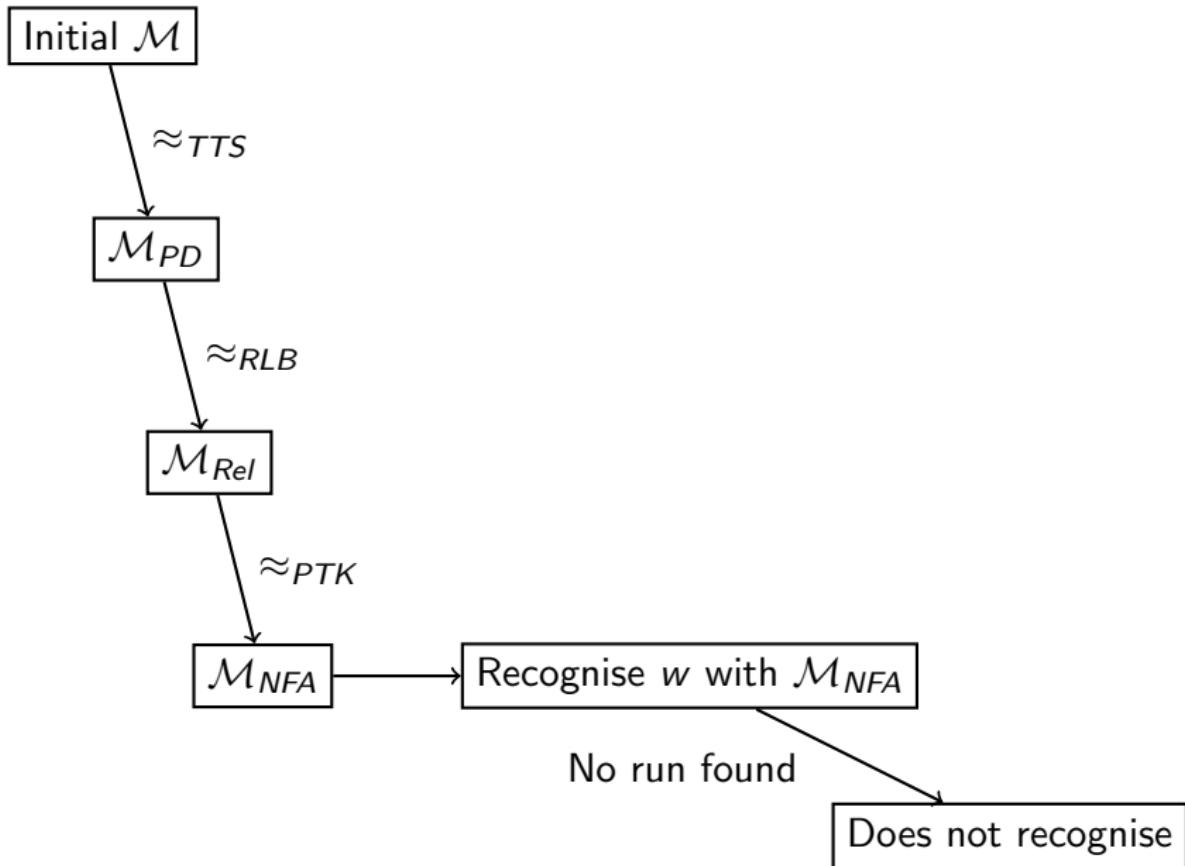
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6: while  $|P_f| < n$  or  $\max_{\theta \in P_f} wt(\theta) > \min_{\theta' \in P_m} wt(\approx_A^{-1}(\theta'))$  do
7:    $\theta_m \leftarrow$  smallest element of  $P_m$ 
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9:   for  $\theta_{m-1} \in \approx_{A_m}^{-1}(\theta_m)$  do
10:    if  $\theta_{m-1} \in R_{\mathcal{M}_{m-1}}$  then
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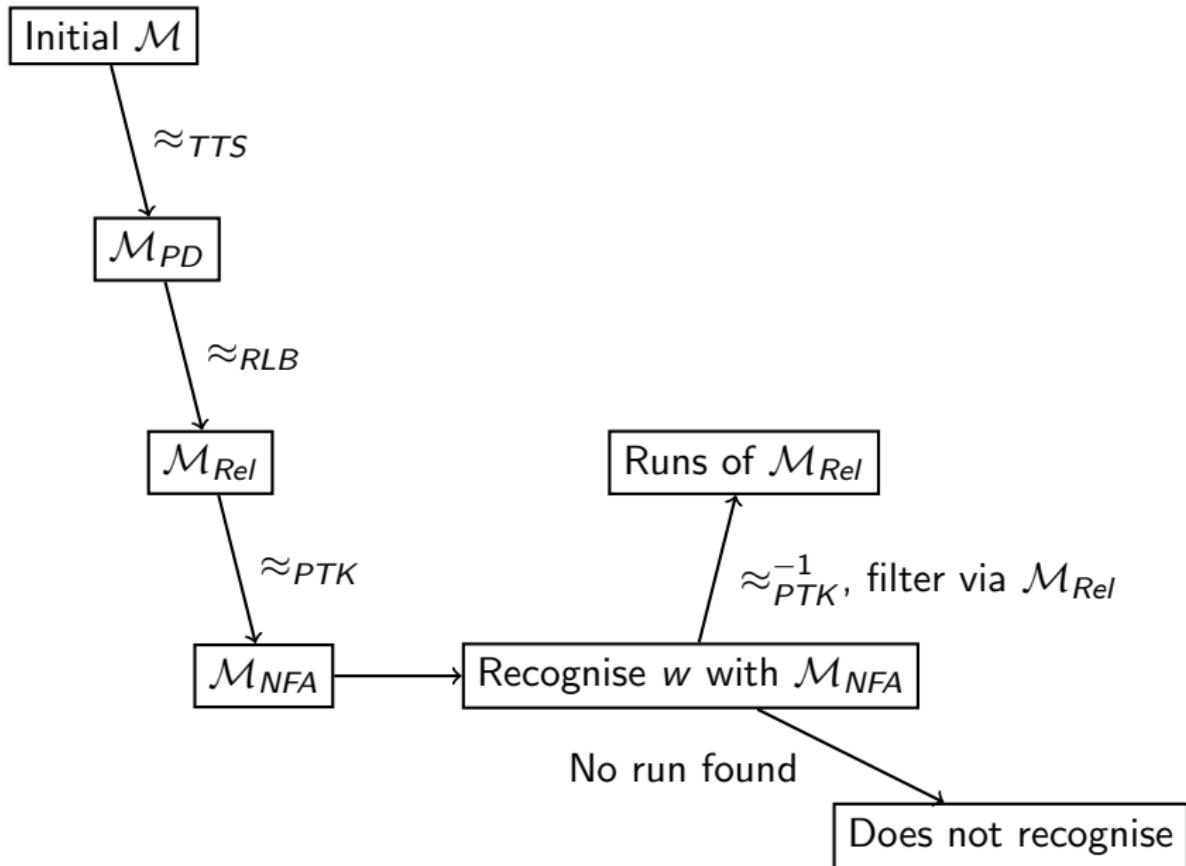
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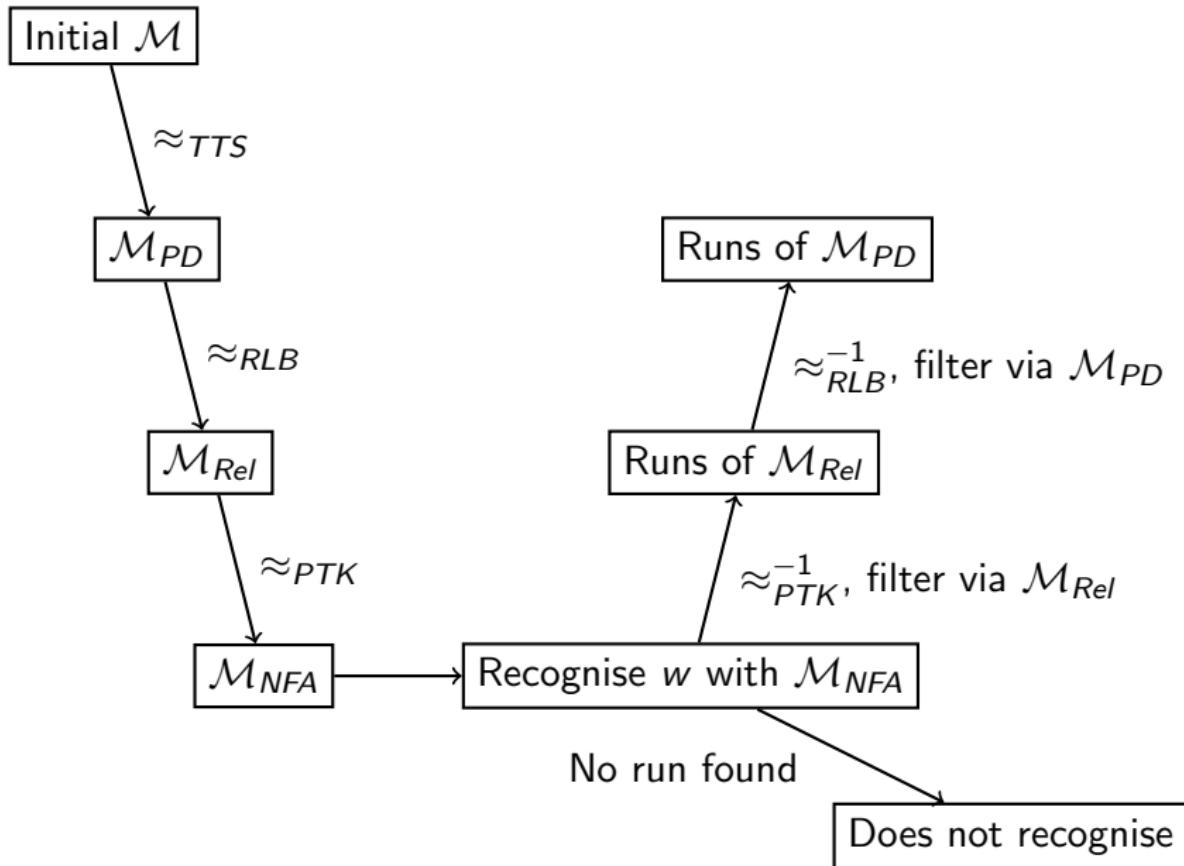
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6: while  $|P_f| < n$  or  $\max_{\theta \in P_f} wt(\theta) > \min_{\theta' \in P_m} wt(\approx_A^{-1}(\theta'))$  do
7:    $\theta_m \leftarrow$  smallest element of  $P_m$ 
8:    $P_m \leftarrow P_m \setminus \{\theta_m\}$ 
9:   for  $\theta_{m-1} \in \approx_{A_m}^{-1}(\theta_m)$  do
10:    if  $\theta_{m-1} \in R_{\mathcal{M}_{m-1}}$  then
11:      for  $\theta_{m-2} \in \approx_{A_{m-1}}^{-1}(\theta_{m-1})$  do
12:        if  $\theta_{m-2} \in R_{\mathcal{M}_{m-2}}$  then
15: return  $P_f$ 
```

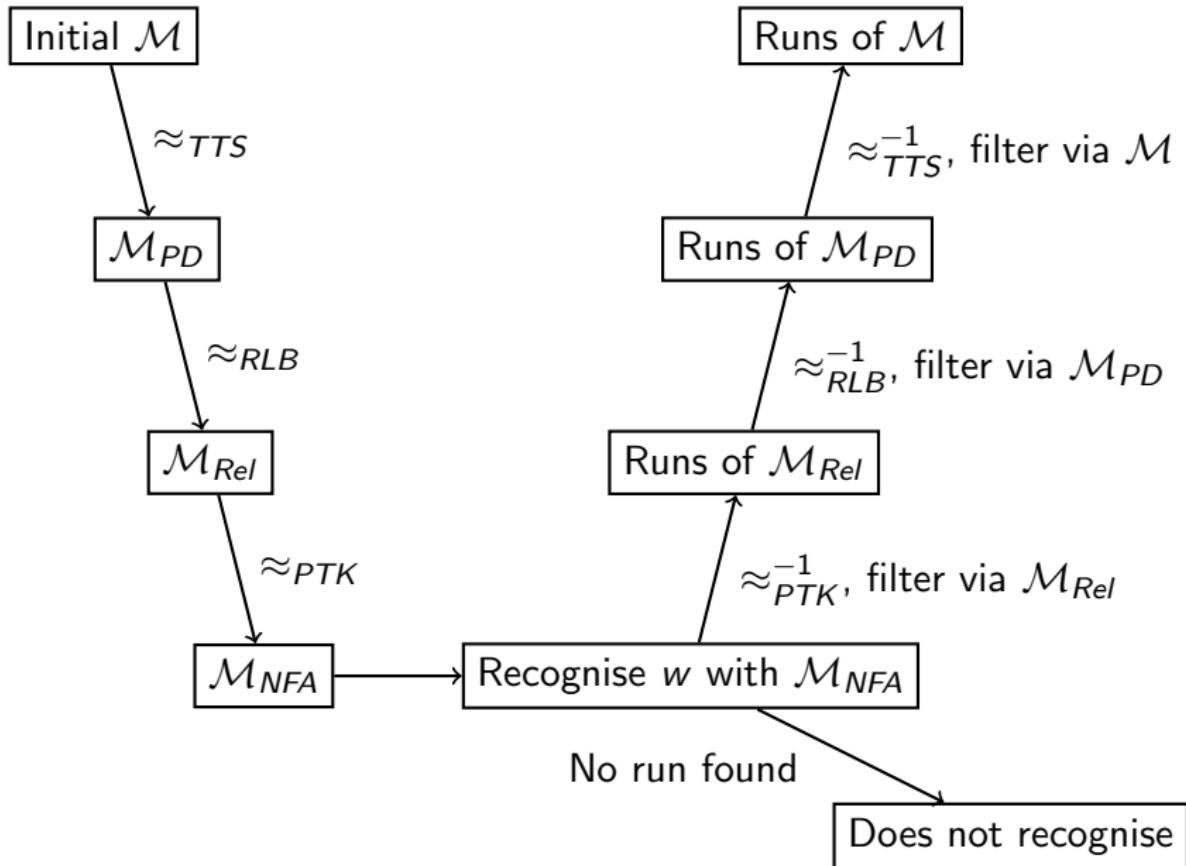
## Algorithm for multiple layers

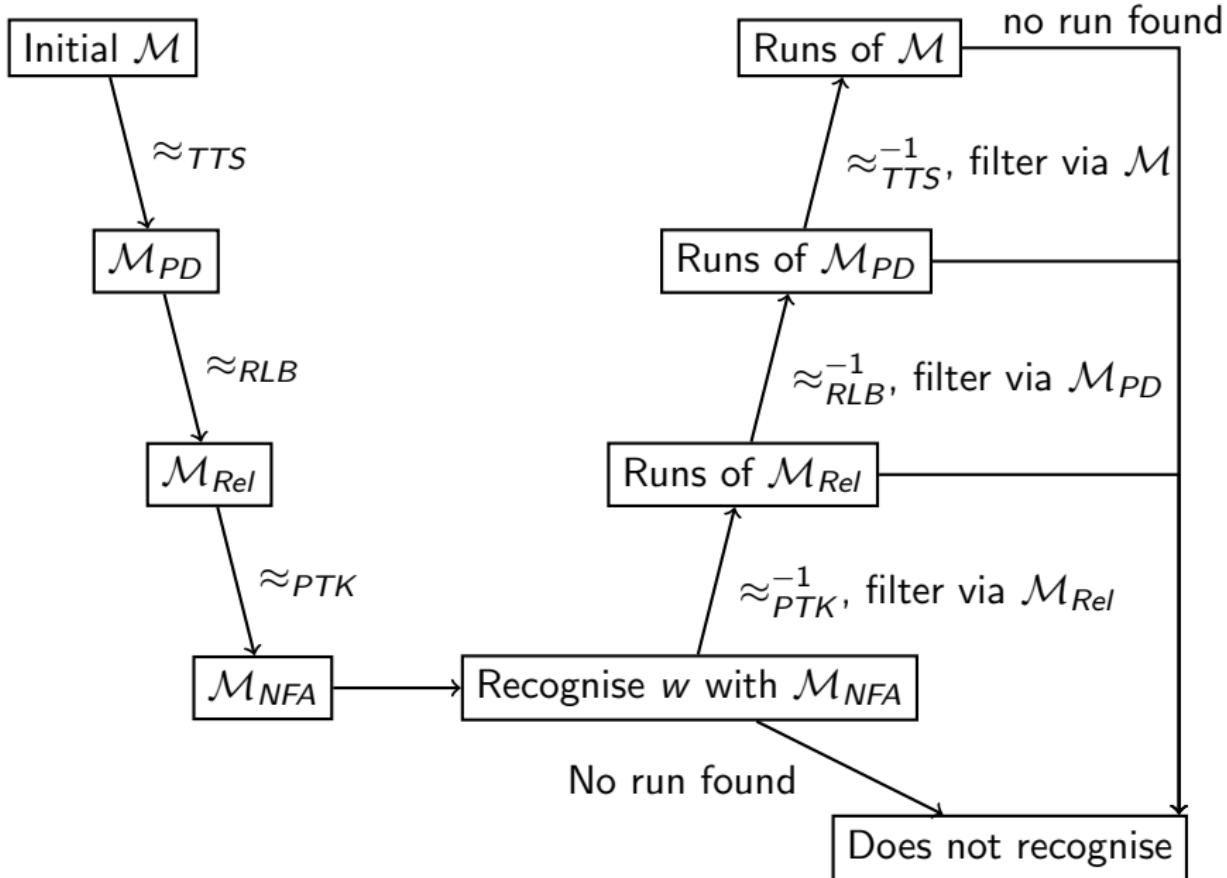
```
1:  $\mathcal{M}_1 \leftarrow \approx_{A_1}(\mathcal{M})$ 
2:  $\mathcal{M}_2 \leftarrow \approx_{A_2}(\mathcal{M}_1)$ 
...
3:  $\mathcal{M}_m \leftarrow \approx_{A_m}(\mathcal{M}_{m-1})$ 
4:  $P_f \leftarrow \emptyset$ 
5:  $P_m \leftarrow R_{\mathcal{M}_m}(w)$ 
6: while  $|P_f| < n$  or  $\max_{\theta \in P_f} wt(\theta) > \min_{\theta' \in P_m} wt(\approx_A^{-1}(\theta'))$  do
7:    $\theta_m \leftarrow$  smallest element of  $P_m$ 
8:    $P_m \leftarrow P_m \setminus \{\theta_m\}$ 
9:   for  $\theta_{m-1} \in \approx_{A_m}^{-1}(\theta_m)$  do
10:    if  $\theta_{m-1} \in R_{\mathcal{M}_{m-1}}$  then
11:      for  $\theta_{m-2} \in \approx_{A_{m-1}}^{-1}(\theta_{m-1})$  do
12:        if  $\theta_{m-2} \in R_{\mathcal{M}_{m-2}}$  then
...
13:          for  $\theta_0 \in \approx_{A_1}^{-1}(\theta_1)$  do
14:            if  $\theta_0 \in R_{\mathcal{M}}$  then  $P_f \leftarrow P_f \cup \{\theta_0\}$ 
15: return  $P_f$ 
```











# Outline

# Implementation<sup>1</sup>

---

<sup>1</sup>using *rustomata* <https://github.com/tud-fop/rustomata>

# Implementation<sup>1</sup>

Preprocessing: app0  $\xrightarrow{\text{tts}}$  app1  $\xrightarrow{\text{rlb}}$  app2  $\xrightarrow{\text{ptk}}$  app3

---

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# Implementation<sup>1</sup>

Preprocessing: app0  $\xrightarrow{\text{tts}}$  app1  $\xrightarrow{\text{rlb}}$  app2  $\xrightarrow{\text{ptk}}$  app3

```
for run1 in app3.recognise(word).take(n) {
    let trans_runs1 = ctf_level(run1, &ptk, &app2);
    for run2 in trans_runs1 {
        let trans_runs2 = ctf_level(run2, &rlb, &app1);
        for run3 in trans_runs2 {
            let trans_runs3 = ctf_level(run3, &tts, &app0);
            for run4 in trans_runs3 {
                println!("{:?}", run4);
            }
        }
    }
}
```

---

<sup>1</sup>using *rustomata* <https://github.com/tud-fop/rustumata>

# Outline

## Experiments

- ▶ Grammars created by using the first 5, 10, 15 and 20 sentences of the NEGRA corpus<sup>2</sup>

---

<sup>2</sup><http://www.coli.uni-saarland.de/projects/sfb378/negra-corpus/negra-corpus.html>

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- ▶ recognising three sentences contained in the corresponding corpus

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$$\mathcal{M} \xrightarrow[TTS]{} \mathcal{M}_{PD} \xrightarrow[RLB]{} \mathcal{M}_{Rel} \xrightarrow[PTK]{} \mathcal{M}_{NFA}$$

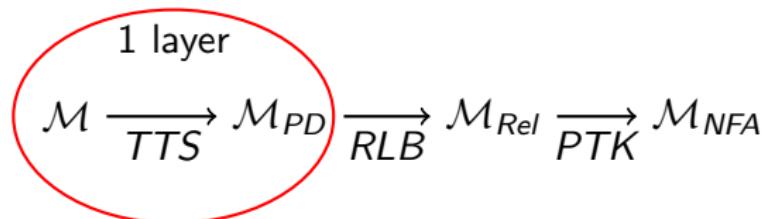
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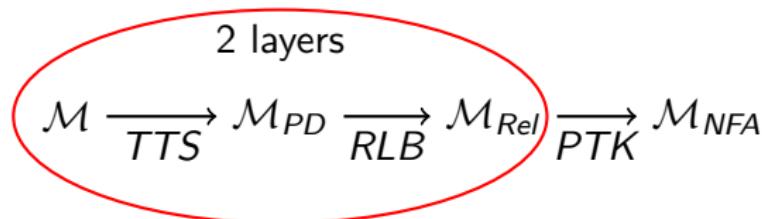
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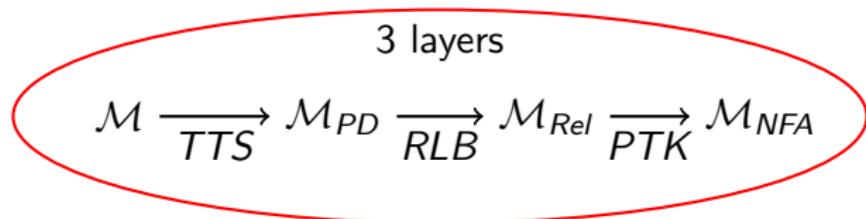


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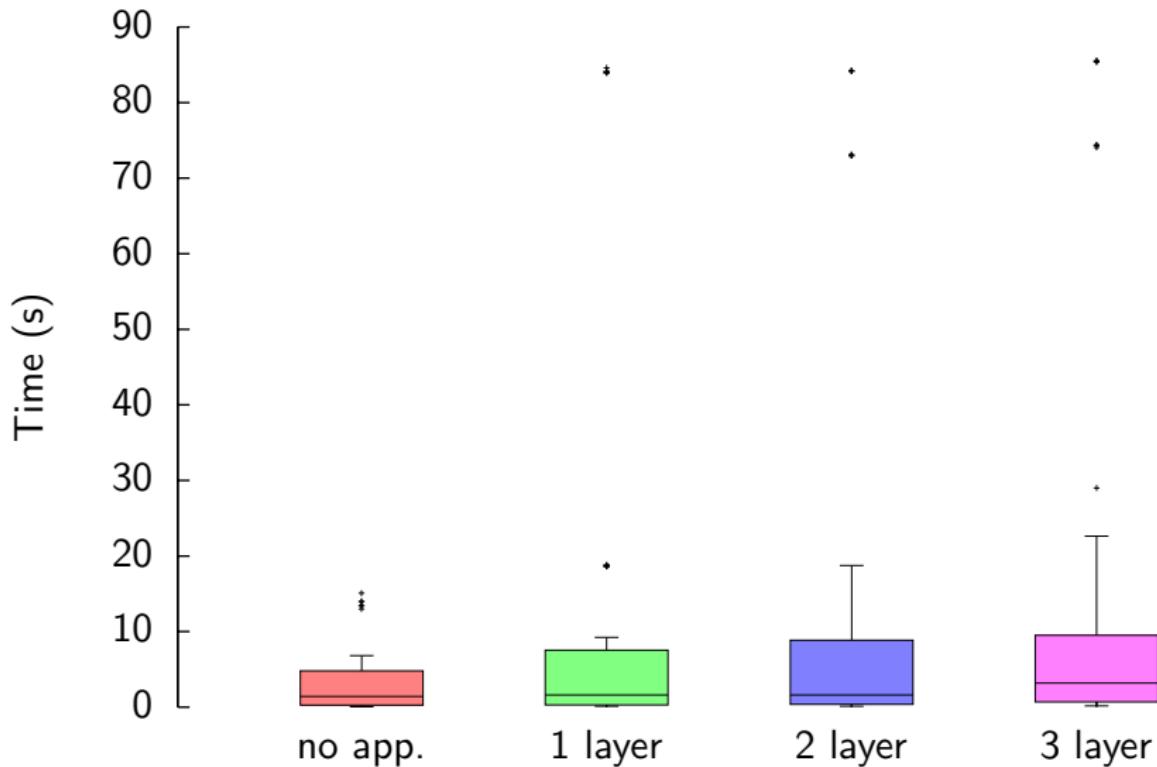


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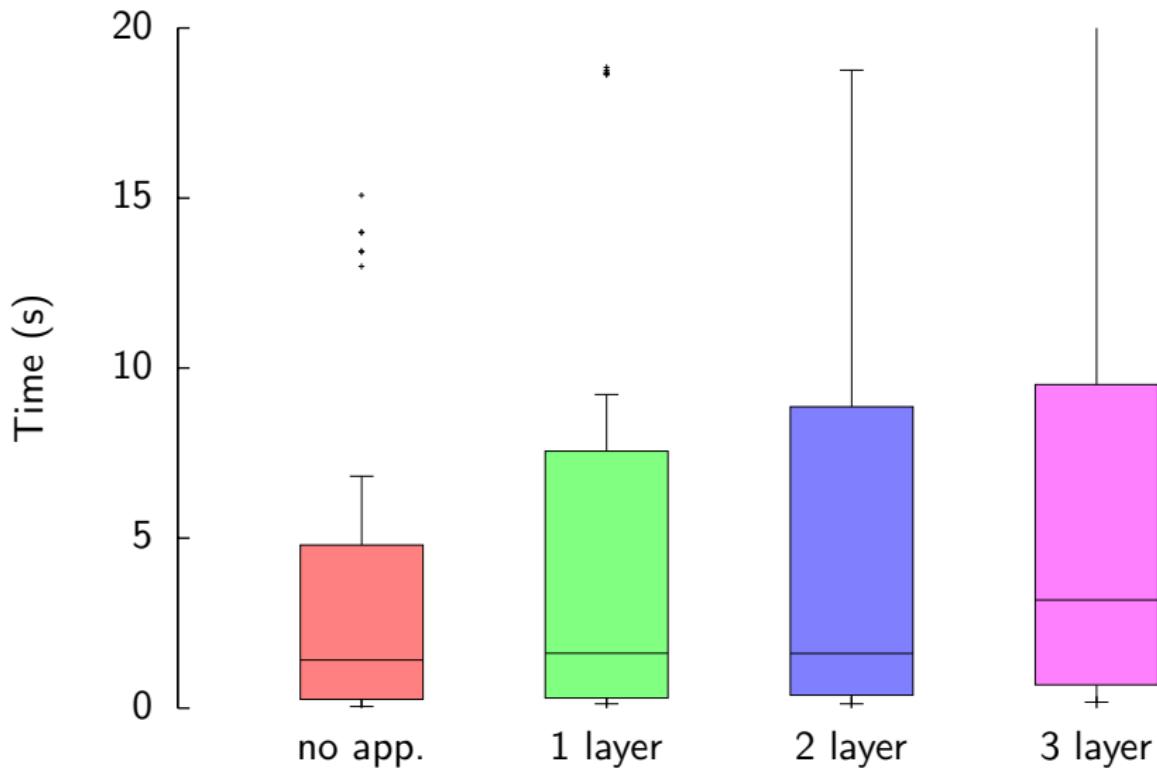
<sup>2</sup><http://www.coli.uni-saarland.de/projects/sfb378/negra-corpus/negra-corpus.html>

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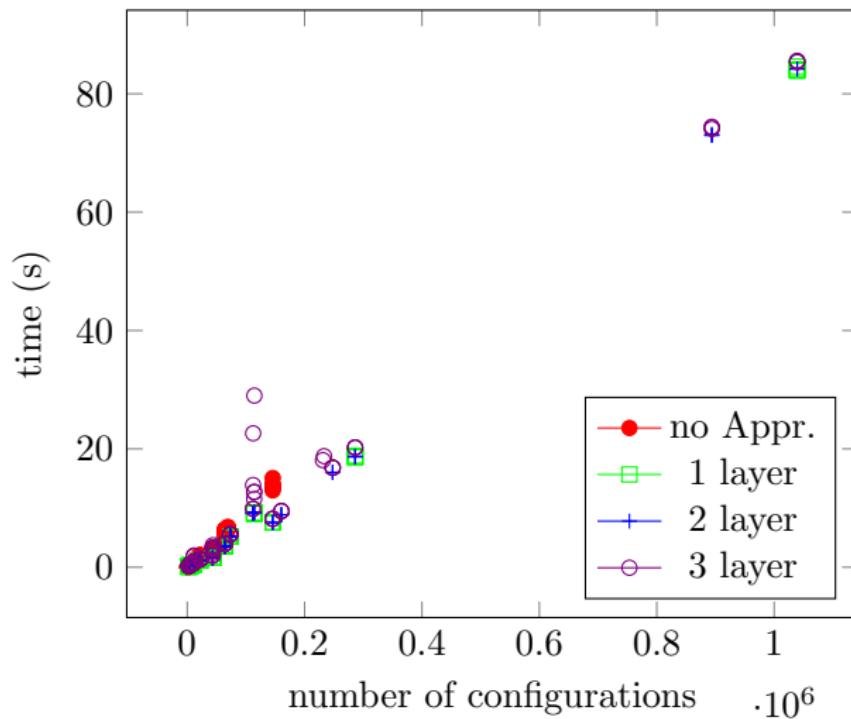
## Results



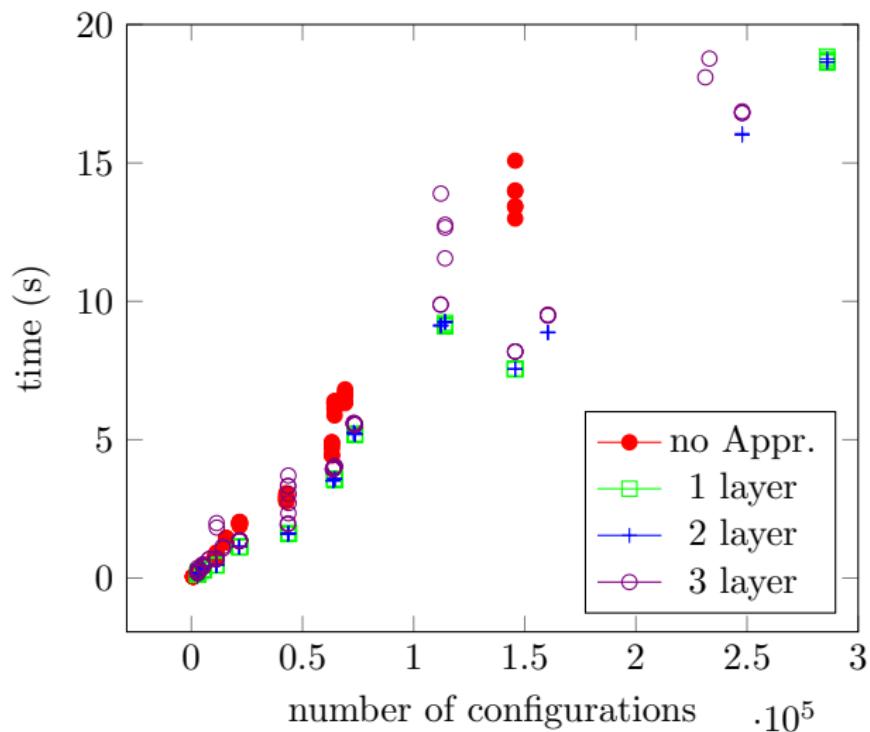
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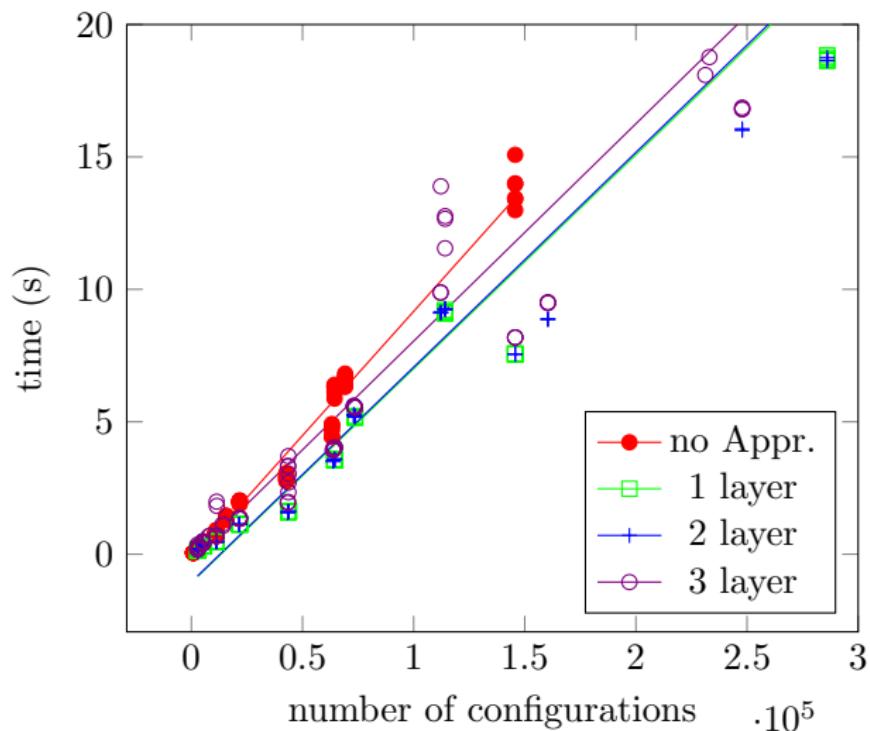
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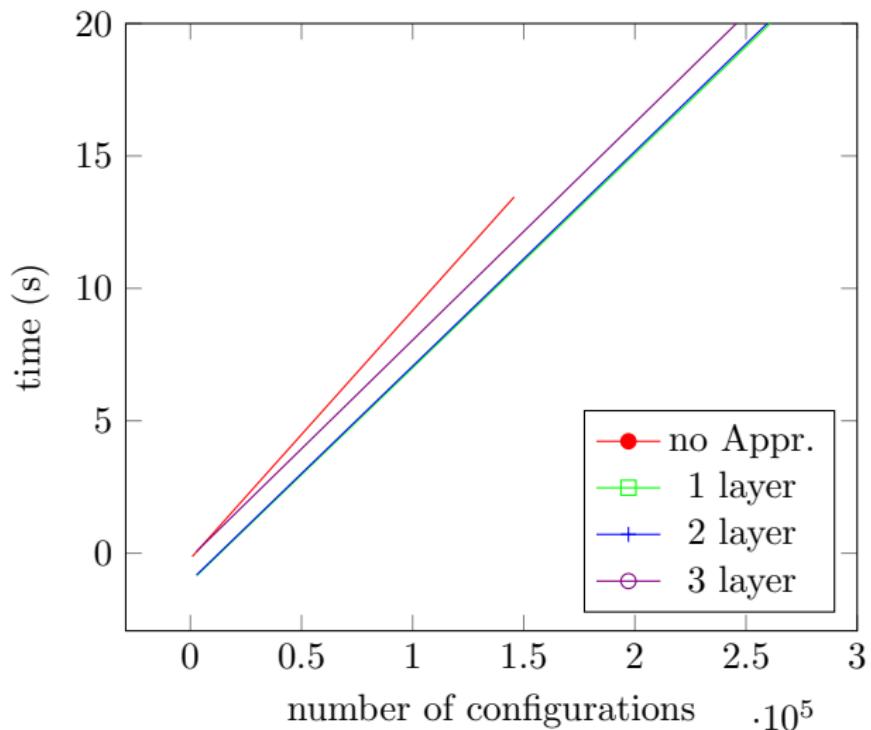
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## References

- [1] Håkan Burden and Peter Ljunglöf. "Parsing Linear Context-Free Rewriting Systems". In: *Proceedings of the Ninth IWPT* (2005), pp. 11–17.
- [2] Eugene Charniak et al. "Multilevel coarse-to-fine PCFG parsing". In: *Proceedings of the HLT-NACL*. 2006.
- [3] Andreas van Cranenburgh. "Efficient Parsing with Linear Context-Free Rewriting Systems". In: *Proceedings of the 13th Conference of the EACL*. (2012), pp. 460–470.
- [4] Tobias Denkinger. "Approximation of Weighted Automata with Storage". In: *Proceedings Eighth International Symposium on GandALF*. 2017, pp. 91–105.
- [5] Mark-Jan Nederhof. "Regular approximations of CFLs: A grammatical view". In: *Proceedings of the IWPT* (1997), pp. 159–170.

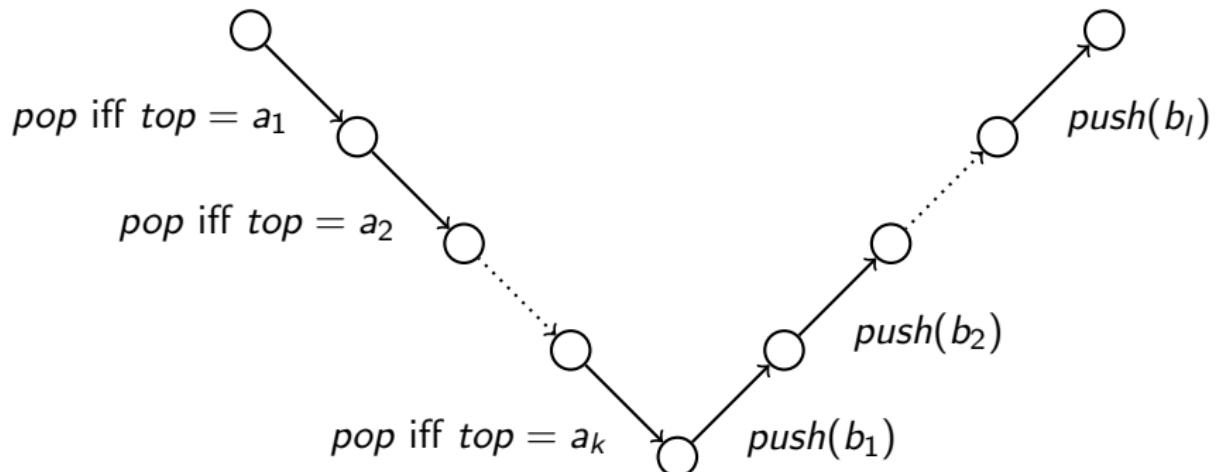
# Stateless Push-Down Automata

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replace( $\underbrace{a_1 a_2 a_3 \dots a_k}_{\text{current symbols}}, \underbrace{b_1 b_2 b_3 \dots b_l}_{\text{new symbols}}$ )

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- ▶  $push(\gamma) = replace(\varepsilon, \gamma)$  for all  $\gamma \in \Gamma$

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## Stateless Tree-Stack Automata

Every instruction is preceded and followed by a  $\text{set}(\gamma)$ .

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State behaviour is encoded into stack-symbols.