

Master-Praktikum:  
Parsing linear context-free rewriting systems

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# Overview

- ▶ Weighted LCFRS
- ▶ Range instantiation
- ▶ Weighted deduction systems
- ▶ Parsing with deduction systems
  - ▶ Overview
  - ▶ CYK parser
  - ▶ Naive parser
  - ▶ Active parser
- ▶ Implementation results
- ▶ Conclusion

# Weighted LCFRS [Vijay-Shanker, Weir, and Joshi 1987; Denkinger 2016]

- ▶ string LCFRS as case of MCFG
- ▶ rules of form  $A \rightarrow \langle ax_{1,1}, cx_{1,2} \rangle(A)$ 
  - ▶ composition representation implies function using string substitution
  - ▶ linear and non-deleting composition
- ▶ rule weights  $p_G: P \rightarrow W$ 
  - ▶ semiring of weights
  - ▶ weight of a derivation: product of rule weights

$$p_G \left( \begin{array}{c} A \rightarrow f(A_1, \dots, A_k) \\ \swarrow \quad \downarrow \quad \searrow \\ A_1 \quad \dots \quad A_k \\ \triangle \quad \quad \quad \triangle \end{array} \right) = p_G(A \rightarrow f(A_1, \dots, A_k)) \cdot \prod_{i=1}^k p_G \left( \begin{array}{c} A_i \\ \triangle \end{array} \right)$$

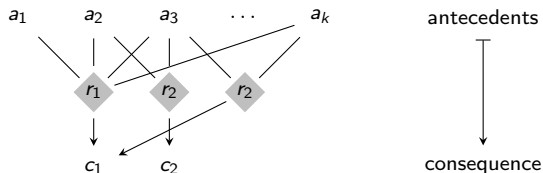
- ▶ weight of a derived word: sum of derivations yielding word

## Ranges and instantiations

- ▶ range: pair of indices limiting subword, e.g.  $0aa_2bccd \rightarrow (0, 2)$
- ▶ range concatenation:  $(i, j)(j, k) = (i, k)$
- ▶ range vector: sequence of non-overlapping ranges
- ▶ instantiated composition: replace terminal sequences with ranges in word
  - ▶  $\langle a_{x_{1,1}}, c_{x_{1,2}} \rangle_{abccd} = \{ \langle (0, 1)_{x_{1,1}}, (3, 4)_{x_{1,2}} \rangle, \langle (0, 1)_{x_{1,1}}, (4, 5)_{x_{1,2}} \rangle, \dots \}$
  - ▶ is ambiguous
  - ▶ implies partial function using range concatenation

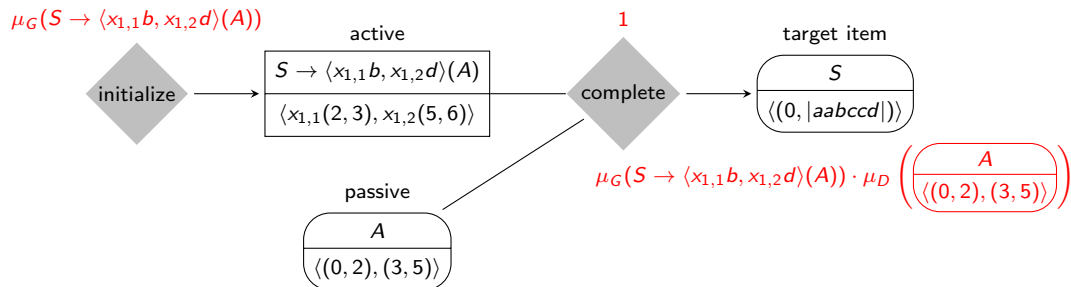
# Weighted deduction [Shieber, Schabes, and Pereira 1995]

- ▶ rule application



- ▶ weighted rules  $\mu_D(r)$
- ▶ weighted items 
$$\mu_{D,C}(c_1) = \mu_D(r_1) \cdot \mu_{D,A}(a_1) \cdot \dots \cdot \mu_{D,A}(a_k) \\ + \mu_D(r_2) \cdot \mu_{D,A}(a_3) \cdot \mu_{D,A}(a_3)(a_k)$$
- ▶ enumerate items using Knuth's algorithm [Nederhof 2003]
- ▶ heuristics [Angelov and Ljunglöf 2014]

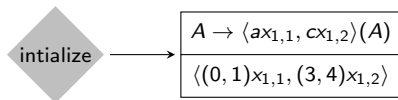
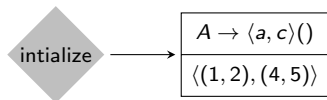
# Deductive parsing: common behavior



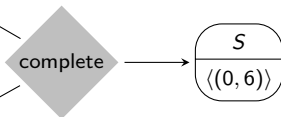
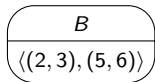
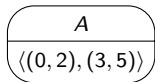
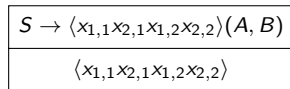
- ▶ parsing items
- ▶ weight paid on initialization
- ▶ completion is free
- ▶ heuristic: approx. cost for target item via inside/outside weights  
[Angelov and Ljunglöf 2014]

# CYK parsing [Seki et al. 1991]

## ► initialize

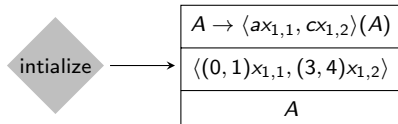
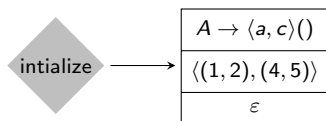


## ► complete

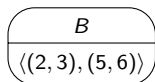
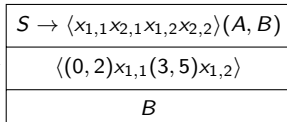
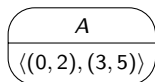
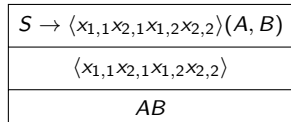


# Naive-active parsing [Burden and Ljunglöf 2005]

► *initialize*



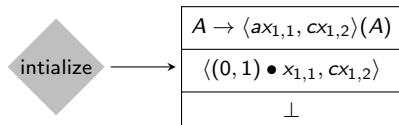
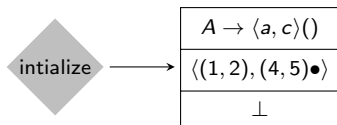
► *complete*



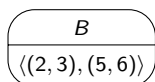
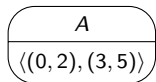
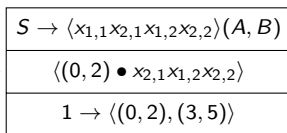
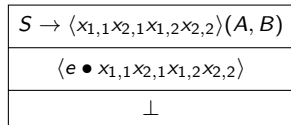


# Active parsing [Burden and Ljunglöf 2005]

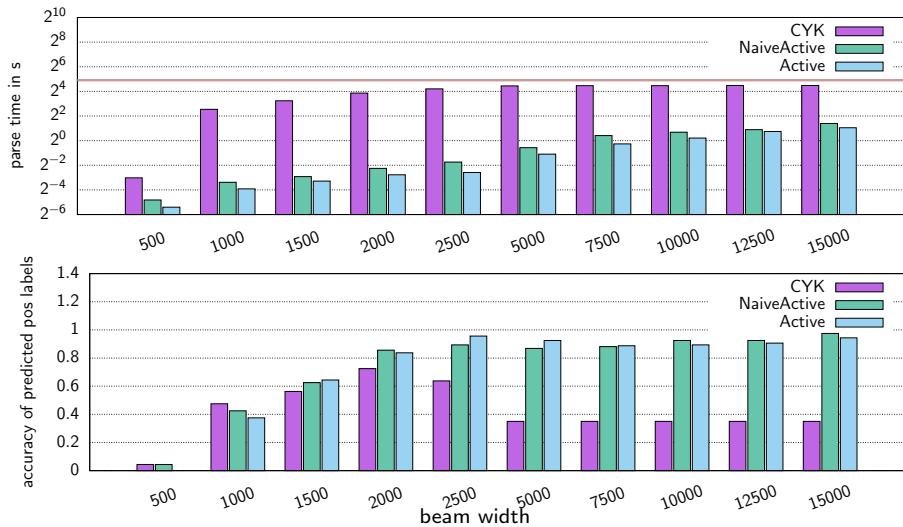
## ▶ initialize



## ▶ complete

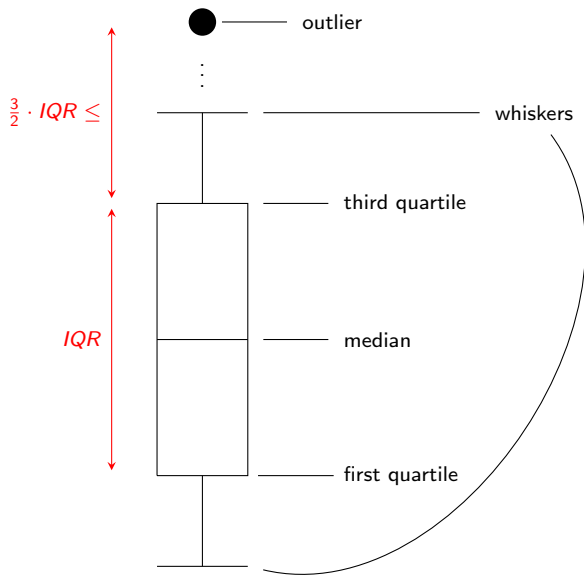


# Results I – Beam search



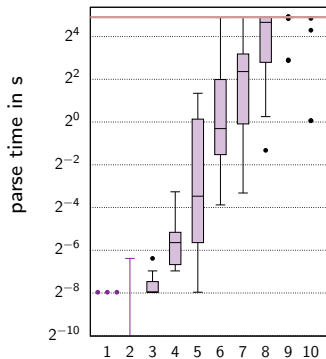
- ▶ average over 20 sentences with 8 tokens
- ▶ grammar extracted from entire NeGra corpus

## Short interlude: box plots

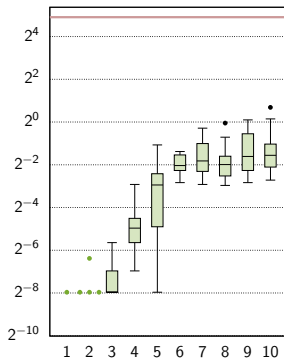


## Results II – parse time for fixed sentence lengths

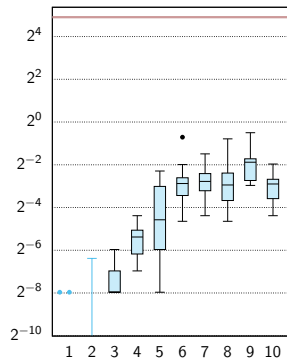
(a) CYK



(b) Naive-active



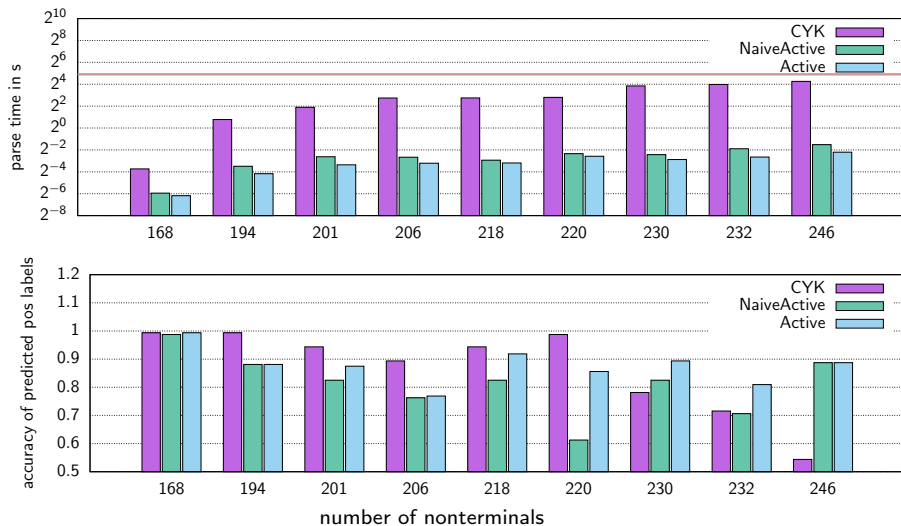
(c) Active



length of sentence to parse, 20 samples for each size

- ▶ average over 20 sentences for each length
- ▶ grammar from entire NeGra corpus
- ▶ beam with 2500

## Results III – Grammar size










- ▶ average over 20 sentences with 8 tokens
- ▶ 9 random sized grammars from NeGra and one from entire corpus
- ▶ beam width 2500

# Conclusions

- ▶ implemented
  - ▶ deduction systems for parsers,
  - ▶ Knuth's algorithm,
  - ▶ three parsersin Vanda
- ▶ used implementation to parse German sentences in NeGra
- ▶ best performance using active parser
- ▶ CYK parser performed significantly worse
- ▶ possible future work:
  - ▶ implement incremental parser
  - ▶ extend implementation for (P)MCFG

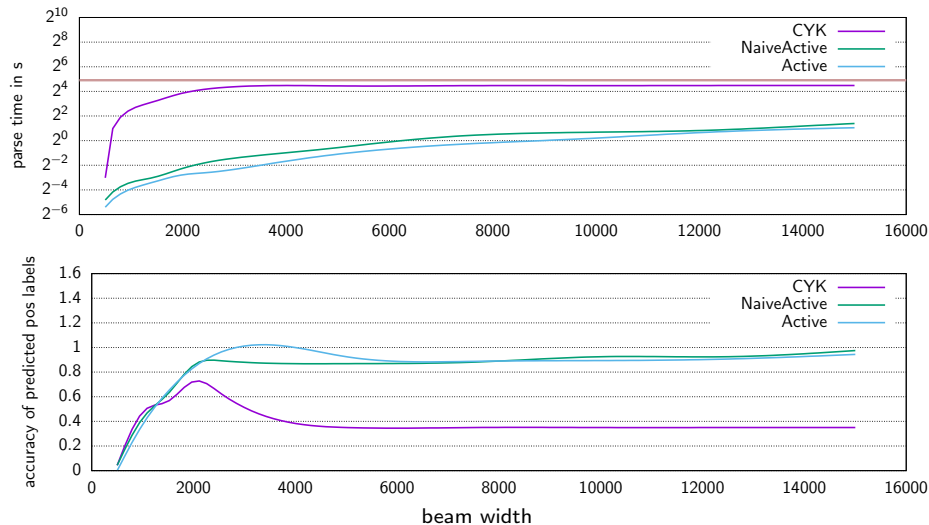
## References

-  Angelov, Krasimir and Peter Ljunglöf (2014). “Fast Statistical Parsing with Parallel Multiple Context-Free Grammars”. In: *Proc. of the 14th Conference of the European Chapter of ACL*.
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-  Nederhof, Mark-Jan (2003). “Weighted deductive parsing and Knuth’s algorithm”. In: *Computational Linguistics* 29.1.
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-  Shieber, Stuart M., Yves Schabes, and Fernando C. N. Pereira (1995). “Principles and Implementation of Deductive Parsing”. In: *J. Log. Program.* 24.1&2.
-  Vijay-Shanker, Krishnamurti, David J Weir, and Aravind K Joshi (1987). “Characterizing structural descriptions produced by various grammatical formalisms”. In: *Proc. of the 25th annual meeting on ACL*. ACL.

BACKUP

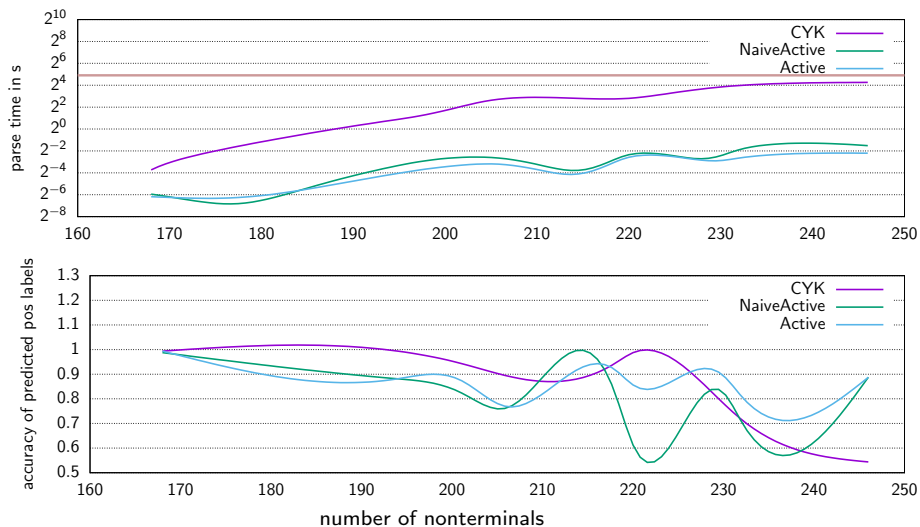


## Results I – Beam search



- ▶ average over 20 sentences with 8 tokens
- ▶ grammar extracted from entire NeGra corpus

## Results III – Grammar size



- ▶ average over 20 sentences with 8 tokens
- ▶ 9 random sized grammars from NeGra and one from entire corpus
- ▶ beam width 2500

# Knuth's Algorithm

```
1 knuth(D = (I, R)):  
2   Chart  $\leftarrow \emptyset$   
3  
4   Agenda  $\leftarrow \{(r(\varepsilon), (r, \varepsilon)) \mid r \in R\}$   
5  
6   while Agenda  $\neq \emptyset$ :  
7     (trigger, backtrace), Agenda  $\leftarrow \text{pop}(\text{Agenda})$   
8  
9     if not trigger  $\in \text{dom}(\text{Chart})$ :  
10      Chart  $\leftarrow \text{Chart}\{\text{trigger}/\text{backtrace}\}$   
11      Agenda  $\leftarrow \text{Agenda} \cup \{(r(\alpha), (r, \alpha)) \mid r \in R, \alpha \in \text{dom}(\text{Chart})^*: \text{trigger in } \alpha\}$   
12    else:  
13      Chart  $\leftarrow \text{Chart}\{\text{trigger}/\text{backtrace}\}$   
14  
15  return Chart
```

# CYK parsing

- ▶ two types rules
  - ▶ initial active items of nonterminals

$$\overline{[A \rightarrow f(A_1, \dots, A_k), \phi]} \quad \forall A \rightarrow f(A_1, \dots, A_k) \in P, \phi \in f_w$$

- ▶ complete all nonterminals at once using passive items

$$\frac{[A \rightarrow f(A_1, \dots, A_k), \phi] (A_1, \rho_1) \dots (A_k, \rho_k)}{(A, \phi(\rho_1, \dots, \rho_k))}$$

- ▶ weights

- ▶  $\mu_{D_{CYK}(G,w)}([A \rightarrow f(A_1, \dots, A_k), \phi]) = p_G(A \rightarrow f(A_1, \dots, A_k))$
- ▶  $\alpha_{D_{CYK}(G,w)}([A \rightarrow f(A_1, \dots, A_k), \phi]) = \alpha_{D_{CYK}(G,w)}((A, \psi)) = \alpha_G(A)$
- ▶  $\beta_{D_{CYK}(G,w)}([A \rightarrow f(A_1, \dots, A_k), \phi]) = \odot_{i=1}^k \beta_G(A_i)$

## CYK parsing - example

$$\begin{aligned}
 & \emptyset \\
 & \vdash_{D_{\text{CYK}}(G,w)} \left\{ \left[ \begin{array}{c} A \rightarrow \langle \varepsilon, \varepsilon \rangle \\ \langle \varepsilon, \varepsilon \rangle \end{array} \right], \left[ \begin{array}{c} A \rightarrow \langle ax_{1,1}, cx_{1,2} \rangle (A) \\ \langle (0, 1)x_{1,1}, (2, 3)x_{1,2} \rangle \end{array} \right], \left[ \begin{array}{c} B \rightarrow \langle \varepsilon, \varepsilon \rangle \\ \langle \varepsilon, \varepsilon \rangle \end{array} \right] \right. \\
 & \quad \left. , \left[ \begin{array}{c} B \rightarrow \langle bx_{1,1}, dx_{1,2} \rangle (B) \\ \langle (1, 2)x_{1,1}, (3, 4)x_{1,2} \rangle \end{array} \right], \left[ \begin{array}{c} S \rightarrow \langle x_{1,1}x_{2,1}x_{1,2}x_{2,2} \rangle (A, B) \\ \langle x_{1,1}x_{2,1}x_{1,2}x_{2,2} \rangle \end{array} \right], \dots \right\} \\
 & \vdash_{D_{\text{CYK}}(G,w)} \left\{ \dots, \left( \begin{array}{c} A \\ \langle \varepsilon, \varepsilon \rangle \end{array} \right), \left( \begin{array}{c} B \\ \langle \varepsilon, \varepsilon \rangle \end{array} \right), \dots \right\} \\
 & \vdash_{D_{\text{CYK}}(G,w)} \left\{ \dots, \left( \begin{array}{c} A \\ \langle (0, 1), (2, 3) \rangle \end{array} \right), \left( \begin{array}{c} B \\ \langle (1, 2), (3, 4) \rangle \end{array} \right), \left( \begin{array}{c} S \\ \langle \varepsilon, \varepsilon \rangle \end{array} \right), \dots \right\} \\
 & \vdash_{D_{\text{CYK}}(G,w)} \left\{ \dots, \left( \begin{array}{c} S \\ \langle (0, 4) \rangle \end{array} \right), \dots \right\}
 \end{aligned}$$

## CYK parsing (optimized) - example

$$\begin{aligned}
 & \emptyset \\
 & \vdash_{D_{CYK}(G,w)} \left\{ \left( \begin{array}{c} A \\ \langle e_1, e_2 \rangle \end{array} \right), \left[ A \rightarrow \langle ax_{1,1}, cx_{1,2} \rangle (A) \right], \left( \begin{array}{c} B \\ \langle e_3, e_4 \rangle \end{array} \right) \right. \\
 & \quad \left. , \left[ B \rightarrow \langle bx_{1,1}, dx_{1,2} \rangle (B) \right], \left[ S \rightarrow \langle x_{1,1}x_{2,1}x_{1,2}x_{2,2} \rangle (A, B) \right], \dots \right\} \\
 & \quad \left. , \left[ \langle (1, 2)x_{1,1}, (3, 4)x_{1,2} \rangle \right], \left[ \begin{array}{c} \langle x_{1,1}x_{2,1}x_{1,2}x_{2,2} \rangle \end{array} \right], \dots \right\} \\
 & \vdash_{D_{CYK}(G,w)} \left\{ \dots, \left( \begin{array}{c} A \\ \langle (0, 1), (2, 3) \rangle \end{array} \right), \left( \begin{array}{c} B \\ \langle (1, 2), (3, 4) \rangle \end{array} \right), \left( \begin{array}{c} S \\ \langle e_5 \rangle \end{array} \right), \dots \right\} \\
 & \vdash_{D_{CYK}(G,w)} \left\{ \dots, \left( \begin{array}{c} S \\ \langle (0, 4) \rangle \end{array} \right), \dots \right\}
 \end{aligned}$$

# Naive parsing

- ▶ three types rules
  - ▶ initial active items of nonterminals

$$\overline{[A \rightarrow f(A_1, \dots, A_k), \phi, A_1 \dots A_k]} \quad \forall A \rightarrow f(A_1, \dots, A_k) \in P, \forall \phi \in f_w$$

- ▶ complete one nonterminal using a passive item

$$\frac{[A \rightarrow f(A_1, \dots, A_k), \phi, A_i \dots A_k](A_i, \rho_i)}{[A \rightarrow f(A_1, \dots, A_k), \phi(\rho_i), A_{i+1} \dots A_k]}$$

- ▶ convert a completed active item

$$\frac{[A \rightarrow f(A_1, \dots, A_k), \rho, \varepsilon]}{(A, \rho)}$$

- ▶ weights

- ▶  $\mu_{D_{CYK}(G,w)}([A \rightarrow f(A_1, \dots, A_k), \phi, A_i \dots A_k]) = \mu_G(A \rightarrow f(A_1, \dots, A_k)) \odot \bigodot_{j=1}^{i-1} \mu_{D_{CYK}(G,w)}((A_j, \rho_j))$   
if the item was previously completed with  $(A_j, \rho_j)$  for each  $j \in [i]$
- ▶  $\alpha_{D_{CYK}(G,w)}([A \rightarrow f(A_1, \dots, A_k), \phi, A_i \dots A_k]) = \alpha_{D_{CYK}(G,w)}((A, \rho) = \alpha_G(A)$
- ▶  $\beta_{D_{CYK}(G,w)}([A \rightarrow f(A_1, \dots, A_k), \phi, A_i \dots A_k]) = \bigodot_{j=i}^k \beta_G(A_j)$

## Naive parsing – example

$$\begin{aligned}
 & \emptyset \\
 \vdash_{D_{NA}(G,w)} & \left\{ \left[ \begin{array}{c} A \rightarrow \langle \varepsilon, \varepsilon \rangle () \\ \langle e_1, e_2 \rangle \\ \varepsilon \end{array} \right], \left[ \begin{array}{c} A \rightarrow \langle ax_{1,1}, cx_{1,2} \rangle (A) \\ \langle (0, 1)x_{1,1}, (2, 3)x_{1,2} \rangle \\ A \end{array} \right], \left[ \begin{array}{c} B \rightarrow \langle \varepsilon, \varepsilon \rangle () \\ \langle e_3, e_4 \rangle \\ \varepsilon \end{array} \right] \\
 & \left. , \left[ \begin{array}{c} B \rightarrow \langle bx_{1,1}, dx_{1,2} \rangle (B) \\ \langle (1, 2)x_{1,1}, (3, 4)x_{1,2} \rangle \\ B \end{array} \right], \left[ \begin{array}{c} S \rightarrow \langle x_{1,1}x_{2,1}x_{1,2}x_{2,2} \rangle (A, B) \\ \langle x_{1,1}x_{2,1}x_{1,2}x_{2,2} \rangle \\ AB \end{array} \right], \dots \right\} \\
 \vdash_{D_{NA}(G,w)} & \left\{ \dots, \left( \begin{array}{c} A \\ \langle e_1, e_2 \rangle \end{array} \right), \left( \begin{array}{c} B \\ \langle e_3, e_4 \rangle \end{array} \right), \dots \right\} \\
 \vdash_{D_{NA}(G,w)} & \left\{ \left[ \begin{array}{c} A \rightarrow \langle ax_{1,1}, cx_{1,2} \rangle (A) \\ \langle (0, 1), (2, 3) \rangle \\ \varepsilon \end{array} \right], \left[ \begin{array}{c} B \rightarrow \langle bx_{1,1}, dx_{1,2} \rangle (B) \\ \langle (1, 2), (3, 4) \rangle \\ \varepsilon \end{array} \right], \left[ \begin{array}{c} S \rightarrow \langle x_{1,1}x_{2,1}x_{1,2}x_{2,2} \rangle (A, B) \\ \langle x_{2,1}x_{2,2} \rangle \\ B \end{array} \right], \dots \right\} \\
 \vdash_{D_{NA}(G,w)} & \left\{ \dots, \left( \begin{array}{c} A \\ \langle (0, 1), (2, 3) \rangle \end{array} \right), \left( \begin{array}{c} B \\ \langle (1, 2), (3, 4) \rangle \end{array} \right), \left( \begin{array}{c} S \\ \langle e_5 \rangle \end{array} \right), \dots \right\} \\
 \vdash_{D_{NA}(G,w)} & \left\{ \dots, \left[ \begin{array}{c} S \rightarrow \langle x_{1,1}x_{2,1}x_{1,2}x_{2,2} \rangle (A, B) \\ \langle (0, 1)x_{2,1}(2, 3)x_{2,2} \rangle \\ B \end{array} \right], \dots \right\} \\
 \vdash_{D_{NA}(G,w)} & \left\{ \dots, \left( \begin{array}{c} S \\ \langle (0, 4) \rangle \end{array} \right), \dots \right\}
 \end{aligned}$$



## Naive parsing (optimized) - example

$$\begin{aligned}
 & \emptyset \\
 & \vdash_{D_{NA}(G,w)} \left\{ \left( \begin{array}{c} A \\ \langle e_1, e_2 \rangle \end{array} \right), \left[ \begin{array}{c} A \rightarrow \langle ax_{1,1}, cx_{1,2} \rangle (A) \\ \langle (0,1)x_{1,1}, (2,3)x_{1,2} \rangle \\ A \end{array} \right], \left( \begin{array}{c} B \\ \langle e_3, e_4 \rangle \end{array} \right) \right. \\
 & \quad \left. , \left[ \begin{array}{c} B \rightarrow \langle bx_{1,1}, dx_{1,2} \rangle (B) \\ \langle (1,2)x_{1,1}, (3,4)x_{1,2} \rangle \\ B \end{array} \right], \left[ \begin{array}{c} S \rightarrow \langle x_{1,1}x_{2,1}x_{1,2}x_{2,2} \rangle (A, B) \\ \langle x_{1,1}x_{2,1}x_{1,2}x_{2,2} \rangle \\ AB \end{array} \right], \dots \right\} \\
 & \vdash_{D_{NA}(G,w)} \left\{ \dots \left( \begin{array}{c} A \\ \langle (0,1), (2,3) \rangle \end{array} \right), \left( \begin{array}{c} B \\ \langle (1,2), (3,4) \rangle \end{array} \right), \left[ \begin{array}{c} S \rightarrow \langle x_{1,1}x_{2,1}x_{1,2}x_{2,2} \rangle (A, B) \\ \langle x_{2,1}x_{2,2} \rangle \\ B \end{array} \right], \dots \right\} \\
 & \vdash_{D_{NA}(G,w)} \left\{ \dots, \left( \begin{array}{c} S \\ \langle e_5 \rangle \end{array} \right), \left[ \begin{array}{c} S \rightarrow \langle x_{1,1}x_{2,1}x_{1,2}x_{2,2} \rangle (A, B) \\ \langle (0,1)x_{2,1}(2,3)x_{2,2} \rangle \\ B \end{array} \right], \dots \right\} \\
 & \vdash_{D_{NA}(G,w)} \left\{ \dots, \left( \begin{array}{c} S \\ \langle (0,4) \rangle \end{array} \right), \dots \right\}
 \end{aligned}$$

# Active parsing I

## ▶ rules

- ▶ initial active items of nonterminals

$$\frac{}{[A \rightarrow f(A_1, \dots, A_k), \langle e \bullet f \rangle, \perp]} \forall A \rightarrow f(A_1, \dots, A_k) \in P$$

- ▶ read terminal

$$\frac{[A \rightarrow f(A_1, \dots, A_k), \langle \psi, r \bullet \sigma \phi \rangle, \Gamma]}{[A \rightarrow f(A_1, \dots, A_k), \langle \psi, (r \cdot r_\sigma) \bullet \phi \rangle, \Gamma]}$$

- ▶ read component of non-completed nonterminal

$$\frac{[A \rightarrow f(A_1, \dots, A_k), \langle \psi, r \bullet x_{i,j} \phi \rangle, \Gamma] (A_i, \rho_i)}{[A \rightarrow f(A_1, \dots, A_k), \langle \psi, (r \cdot \rho_{i,j}) \bullet \phi \rangle, \Gamma[i/\rho]}$$

- ▶ read component of known nonterminal

$$\frac{[A \rightarrow f(A_1, \dots, A_k), \langle \psi, r \bullet x_{i,j} \phi \rangle, \Gamma]}{[A \rightarrow f(A_1, \dots, A_k), \langle \psi, (r \cdot \Gamma(i)_j) \bullet \phi \rangle, \Gamma]}$$

- ▶ convert a completed active item

$$\frac{[A \rightarrow f(A_1, \dots, A_k), \langle \psi \bullet \varepsilon \rangle, \Gamma]}{(A, \psi, )}$$

# Active parsing II

## ► weights

- $\mu_{D_{ACT}(G,w)}([A \rightarrow f(A_1, \dots, A_k), \langle \psi \bullet \phi \rangle, \Gamma]) = \mu_G(A \rightarrow f(A_1, \dots, A_k)) \odot \bigodot_{i \in \text{dom}(\Gamma)} \mu_{D_{ACT}(G,w)}((A_i, \Gamma(i)))$
- $\alpha_{D_{ACT}(G,w)}([A \rightarrow f(A_1, \dots, A_k), \langle \psi \bullet \phi \rangle, \Gamma]) = \alpha_{D_{ACT}(G,w)}((A, \rho)) = \alpha_G(A)$
- $\beta_{D_{ACT}(G,w)}([A(A_1, \dots, A_k) \rightarrow f, \langle \psi \bullet \phi \rangle, \Gamma]) = \bigodot_{i \in [k]: i \notin \text{dom}(\Gamma)} \beta_G(A_i)$

## Active parsing (optimized) – example

$$\begin{aligned}
 & \emptyset \\
 \vdash_{D_{NA}(G,w)} & \left\{ \left( \begin{array}{c} A \\ \langle e_1, e_2 \rangle \end{array} \right), \left[ \begin{array}{c} A \rightarrow \langle ax_{1,1}, cx_{1,2} \rangle (A) \\ \langle (0, 1) \bullet x_{1,1}, cx_{1,2} \rangle \\ \perp \end{array} \right], \left( \begin{array}{c} B \\ \langle e_3, e_4 \rangle \end{array} \right) \right. \\
 & \left. , \left[ \begin{array}{c} B \rightarrow \langle bx_{1,1}, dx_{1,2} \rangle (B) \\ \langle (1, 2) \bullet x_{1,1}, dx_{1,2} \rangle \\ \perp \end{array} \right], \left[ \begin{array}{c} S \rightarrow \langle x_{1,1}x_{2,1}x_{1,2}x_{2,2} \rangle (A, B) \\ \langle e \bullet x_{1,1}x_{2,1}x_{1,2}x_{2,2} \rangle \\ \perp \end{array} \right], \dots \right\} \\
 \vdash_{D_{NA}(G,w)} & \left\{ \dots \left( \begin{array}{c} A \\ \langle (0, 1), (2, 3) \rangle \end{array} \right), \left( \begin{array}{c} B \\ \langle (1, 2), (3, 4) \rangle \end{array} \right), \left[ \begin{array}{c} S \rightarrow \langle x_{1,1}x_{2,1}x_{1,2}x_{2,2} \rangle (A, B) \\ \langle e \bullet x_{2,1}x_{1,2}x_{2,2} \rangle \\ 1 \rightarrow \langle e_1, e_2 \rangle \end{array} \right], \dots \right\} \\
 \vdash_{D_{NA}(G,w)} & \left\{ \dots, \left( \begin{array}{c} S \\ \langle e_5 \rangle \end{array} \right), \left[ \begin{array}{c} S \rightarrow \langle x_{1,1}x_{2,1}x_{1,2}x_{2,2} \rangle (A, B) \\ \langle (0, 1) \bullet x_{2,1}x_{1,2}x_{2,2} \rangle \\ 1 \rightarrow \langle (0, 1), (2, 3) \rangle \end{array} \right], \dots \right\} \\
 \vdash_{D_{NA}(G,w)} & \left\{ \dots, \left( \begin{array}{c} S \\ \langle (0, 4) \rangle \end{array} \right), \dots \right\}
 \end{aligned}$$